Consumption Baskets and Currency Choice in International Borrowing

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Abstract
We propose a novel explanation for the empirically low prevalence of external borrowing in local currency in emerging market economies, a phenomenon sometimes referred to as the \textit{original sin} of international finance. We study the endogenous currency denomination of an emerging economy’s assets and liabilities within the context of a dynamic stochastic general equilibrium (DSGE) model with portfolio choice featuring non-tradable goods and nominal rigidities. We find that these features lower the correlation between the local currency and domestic lenders’ consumption. They thus reduce the risk premium demanded by domestic lenders on local currency debt, and can therefore help explain the low willingness of foreigners to lend in local currency.

\textit{Keywords:} Dollarization, External Debt, Original Sin, Portfolio Choice

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1. Introduction

The existing international finance literature has identified currency mismatch associated with the large prevalence of foreign currency in external borrowing as a key contributor to the vulnerability and volatility of emerging market economies (see e.g., Eichengreen and Hausmann 1999 and Goldstein and Turner 2004). This phenomenon has to some extent been attenuated by the development of local currency bond markets in recent years, but it remains pervasive across most emerging and developing countries. A number of explanations have been put forward to account for it. Some point to inherent weaknesses of these emerging economies, while others emphasize factors exogenous to emerging economies, such as the characteristics of the international monetary system. In this paper, we offer a novel explanation that neither relies on fundamental weaknesses, nor on institutional features of international financial markets, but rather on characteristics of goods markets in the domestic economy.

Our theory is framed in the context of a small open emerging economy where both domestic and foreign lenders compete to purchase bonds issued by domestic borrowers. Bonds can be issued in both local and foreign currencies, and the currency composition of assets and liabilities is determined in equilibrium. When measured in terms of the foreign currency, local currency bonds are risky while foreign currency bonds are risk-free. The willingness of borrowers to hold local currency bonds thus depends on how their consumption risk correlates with the value of the local currency. Our main insight relies on the observation that when domestic agents’ consumption baskets comprise non-tradable goods, the sign and magnitude of this correlation crucially depends on the prevailing degree of nominal rigidities.

We consider two sources of shocks in the small open emerging economy: real and nominal. Under flexible prices, money is neutral and consumption risk is only associated with real shocks. Negative real shocks reduce domestic consumption and depreciate the local currency, thereby adversely affecting the payoff on local currency bonds. Thus, as long as foreigners’ consumption is not too correlated with the emerging economy’s country risk, local currency bonds are a worse hedge for domestic lenders than for foreign lenders. As a result, foreign lenders require a lower risk-premium than domestic lenders on local currency bonds, and in equilibrium they are the ones extending local currency denominated credit to domestic borrowers. When domestic prices are sticky, however, nominal shocks impact real allocations and decisively alter the correlation structure between domestic agents’ consumption and the returns on local currency bonds. In particular, positive shocks to the domestic money supply, which as in the flexible price case depreciate the local currency, are now expansionary in the non-tradable sector due to a traditional Keynesian channel and are therefore associated with an increase in domestic lenders’ consumption. Relative to the flexible price case, this channel lowers the correlation between domestic lender’s consumption and the payoff on local

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1 See e.g., Lane and Shambaugh (2010), who document that out of the 93 low and middle income countries present in their dataset, an overwhelming majority have 0% of their foreign debt liabilities denominated in local currencies. Only 4 of them have more than 1%: South Africa (6.5%), Uruguay (2.5%), Thailand (1.8%) and Slovak Republic (1.4%).
currency bonds, and therefore improve the quality of the hedge provided by this instrument to domestic lenders relative to foreign lenders. Nominal rigidities in the non-tradable sector thus give rise to a “consumption basket effect” that reduces the risk-premium domestic lenders demand to hold local currency bonds, and help explain why domestic lenders may partially crowd out foreign lenders on the local currency bond market.

We lay out our argument in a simple DSGE model of a small open emerging economy featuring production and price rigidity in the non-tradable goods sector only. Real shocks take the form of shocks to the economy’s endowment of the tradable good, while nominal shocks are shocks to the money supply. The model features three agents: domestic households, domestic entrepreneurs and foreign lenders. Foreign lenders are subject to consumption risk not perfectly correlated with the two domestic shocks, and as a result the model features incomplete financial markets (i.e., local and foreign currency debt allow an imperfect span). In order to address the challenge posed by computing equilibria in an incomplete market model with multiple assets, we extend (to a setting with more than two agents) the method recently developed by Devereux and Sutherland (2010) and Tille and van Wincoop (2010) to solve for the long run currency choices in incomplete markets models.

Our results indicate that nominal rigidities in the non-tradable goods market can bring about a sizable reduction in the extent to which local currency bonds are used for external borrowing relative to a flexible price benchmark. Under fully flexible prices and an empirically plausible correlation between the emerging economy’s output and foreign output, our model predicts that nearly 60% of steady-state external borrowing is denominated in local currency, with the remaining 40% denominated in foreign currency. For an empirically reasonable degree of price stickiness, these shares respectively become 20% and 80%. This suggests that our “consumption basket effect” can potentially have a significant impact on an emerging economy’s currency denomination of external debt.

2. Related literature

A large volume of empirical literature documents the prevalence of foreign currency denominated instruments in the international debt of emerging economies, a phenomenon often labelled the original sin of international finance (or the international dimension thereof). This phenomenon has been argued to entail costs for countries in terms of fiscal solvency, output volatility, exchange rate volatility and capital flow volatility (Eichengreen et al. 2005b), as well as increased risks of currency and debt crises (Bordo et al. 2010). The empirical literature is divided on the question of the causes of the original sin phenomenon. On one hand, authors such as Hausmann and Panizza (2003), Bordo et al. (2005), Eichengreen et al. (2005a) and Flandreau and Sussman (2005) argue that the causes have more to do with the structure of the international financial system or exogenous events than with features or policies of the affected emerging economies, hence the “original sin” label. On the other hand, Goldstein and Turner (2004), Burger and Warnock (2006) and Claessens et al. (2007) defend the view that institutional weaknesses, such as a lack of monetary credibility, fiscal profligacy and a weak rule of law, can account for cross-sectional variation in the incidence
of the original sin phenomenon.²

The theoretical literature offers a number of explanations for the prevalence of foreign currencies in emerging economies’ borrowing. A classic explanation is the lack of monetary credibility (e.g., Bohn [1990], Calvo [1996], Ize and Levy Yeyati [1998], Ize and Paradó [2002], Chamon and Hausmann [2005], and Jeanne [2005]). McKinnon and Pill [1998], Burnside et al. [2001] and Schneider and Tornell [2004] emphasize the role of moral hazard created by bailout guarantees; Caballero and Krishnamurthy [2003] and Korinek [2007] propose explanations relying on financial market imperfections in the domestic economy; Jeanne [2000] and Broda and Levy Yeyati [2006] stress that foreign-currency debt can arise due to commitment or signaling problems at the level of individual borrowers. In contrast, our paper emphasizes the structure of goods markets. In that sense, it is related to the work of Engel and Matsumoto [2009] and Coeurdacier and Gourinchas [2011]. These papers aim at explaining the equity home bias puzzle in symmetric two-country models, and show that little equity diversification is required when agents can hedge their foreign exchange risk sufficiently using either a forward position in foreign exchange (Engel and Matsumoto [2009]) or bonds (Coeurdacier and Gourinchas [2011]). We adopt a different focus on international debt dollarization in a small open economy setup. By explicitly considering within country heterogeneity (i.e., between domestic borrowers and lenders), we highlight the relative advantage of domestic agents in holding local currency bonds and thereby provide an explanation for dollarization of external debt.

In another related paper, Shi and Xu [2010] offer an explanation for twin dollarization, i.e., the fact that both liabilities and exports are often denominated in foreign currency. They argue that a fixed exchange rate may lead to twin dollarization, while a floating exchange rate may instead lead firms to borrow and set export prices in domestic currency. This is because with higher exchange rate flexibility, exporting firms choose to borrow in domestic currency to avoid exchange rate risk. Thus, their prediction is that under flexible exchange rates, countries should borrow in local currency. Our paper, in contrast, can rationalize the prevalence of foreign currency borrowing even under a flexible exchange rate regime.

The paper is organized as follows: Section 3 describes the model. Section 4 discusses the solution and calibration. Section 5 analyzes a special case where bonds are exogenously denominated in foreign currency. Section 6 analyzes the full model with endogenous currency composition of assets and liabilities. And finally section 7 concludes.

3. Model

In this section we describe a model of a monetary small open economy and a set of foreign lenders. Foreign lenders are small and risk averse. This is a crucial deviation from the literature, a point to which we return later. Foreign lenders have access to bonds in local and foreign currency. The small open economy is populated by a continuum of households.

²Hausmann and Panizza [2003] and Mehl and Reynaud [2005] also find support for the view that institutional factors matter for the domestic dimension of the original sin, defined as countries’ inability borrow domestically at fixed rates and long maturities.
and a continuum of entrepreneurs. Both types of agents consume two goods: a good that can be exchanged with the rest of the world (a tradable good) and a good consumed by domestic agents only (a non-tradable good). Output in the tradable goods sector simply follows a stochastic endowment process. In contrast, output in the non-tradable goods sector results from combining labor supplied by households with a non-traded fixed factor (i.e., capital or land) owned by entrepreneurs. Households derive disutility from labor supply, and all domestic agents derive utility from holding real money balances. Like foreign lenders, all domestic agents have access to markets for bonds in local and foreign currency.

The environment features flexible prices in the tradable goods sector, but nominal price rigidities in the non-tradable goods sector. The law of one price is assumed to hold for the tradable good. Following standard practice in the New Keynesian literature, we adopt a monopolistically competitive structure for the non-tradable goods market. To analyze a situation in which households are net lenders and entrepreneurs are net borrowers despite refraining from modeling capital accumulation, we assume that entrepreneurs discount the future more strongly than households and foreign lenders. As a result, relatively impatient entrepreneurs borrow from relatively patient households and foreign lenders in equilibrium.

### 3.1. Households

Households’ expected lifetime utility is given by:

$$E_0 \sum_{t=0}^{\infty} \psi_{H,t} \left[ \left( \frac{c_{H,t} - \Phi L_t^{1+\nu}}{1+\nu} \right)^{1-\sigma} + \chi \ln \left( \frac{M_{H,t}}{P_t} \right) \right],$$

where $c_{H,t}$ is consumption, $L_t$ is employment, $\frac{M_{H,t}}{P_t}$ is real money balances ($P_t$ is an aggregate price level to be defined below), $\psi_{H,t}$ is a discount factor, $E_0$ is the expectation operator conditional on date 0 information, $\sigma$ is the inverse elasticity of substitution, $\nu$ is the inverse Frisch elasticity of labor supply, and $\Phi$ and $\chi$ are parameters governing the level of labor supply and money holdings, respectively. Preferences of this kind are well-known from the work of Greenwood et al. (1988), and have been widely used in the real business cycle literature, as they provide a better description of consumption and the trade balance for small

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3 The assumption of exogenous endowments in the tradable sector is not crucial for our results. Our mechanism requires production in at least one of the two sectors, but not necessarily in both.

4 This asymmetry in price stickiness across the two sectors is not crucial for our results, but significantly simplifies our analysis by allowing us to refrain from modeling production and pricing decisions in the tradable goods sector. This simplification is warranted by the fact that in practice, prices of services (most of which are non-tradable) adjust significantly more sluggishly than goods prices (see, e.g., Bils and Klenow 2004, Nakamura and Steinsson 2008).

5 Assuming away fluctuations in the foreign currency price of this good, we can thus interpret the local currency price of the tradable good as the domestic nominal exchange rate.

6 This simple technical device to generate the desired direction of inter-temporal trade has been widely used in the literature (see, e.g., Devereux and Sutherland 2009).
The consumption index $c_{H,t}$ is defined as $c_{H,t} \equiv \left( c_{H,t}^T \right)^\gamma \left( c_{H,t}^N \right)^{1-\gamma}$, where $c_{H,t}^T$ and $c_{H,t}^N$ denote the household’s tradable and non-tradable goods consumption, respectively. $\psi_{H,t}$ is an endogenous discount factor that depends on the aggregate (economy-wide) level of households’ consumption and is defined as $\psi_{H,0} = 1$ and $\psi_{H,t} = \beta^t \Pi_{k=0}^{t-1} c_{H,k}^{-\mu}$ for $t \geq 1$. Households face a sequence of budget constraints given by

$$P_t^T c_{H,t}^T + P_t^N c_{H,t}^N + B_{H,t} + f_{H,t} + M_{H,t} = R_t B_{H,t-1} + R^* P_t^T f_{H,t-1} + M_{H,t-1} + W_t L_t + P_t^T y_{H,t} + T_{H,t},$$

where $P_t^T$ and $P_t^N$ are, respectively, the nominal prices of the tradable and non-tradable goods, $W_t$ is the nominal wage, $y_{H,t}$ is the household’s tradable goods endowment, $R^*$ is the (gross, constant and exogenous) nominal return on the foreign currency bond and $R_t$ is the (gross) nominal return of the local currency bond, known as of date $t-1$. We assume throughout that foreign inflation is zero. As a result, $R^*$ is also the real return on the foreign currency bond. $B_{H,t}$ and $f_{H,t}$ are the household’s holdings of local and foreign currency bonds, respectively. Finally, $T_{H,t}$ is a lump-sum transfer used by the government to rebate revenues from money creation. The aggregate price index satisfying $P_t c_{H,t} = P_t^T c_{H,t}^T + P_t^N c_{H,t}^N$ is given by $P_t = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} \left( P_t^T \right)^\gamma \left( P_t^N \right)^{1-\gamma}$ and can be derived from expenditure minimization. Each period, the household receives a wage payment, a tradable goods endowment, a government transfer, as well as returns on past bond and money holdings. It chooses how to spend these proceeds between consumption, bond investment and money holding.

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7 The key analytical simplification introduced by the Greenwood-Hercowitz-Huffman (GHH) preferences is that there is no wealth effect on labor supply.

8 As in Schmitt-Grohe and Uribe (2003), assuming an endogenous discount factor is a simple technical device to induce uniqueness of the model’s deterministic steady state and stationary responses to temporary shocks. Specifically, the uninternalized endogenous discount factor decreases with the aggregate household consumption, which an individual household takes as given. In our calibration, $\mu$ is set very small so that in the short run, the deviations of the endogenous discount factor from the standard discount factor $\beta$ are negligible.

9 Positive values correspond to long position (i.e., lending), while negative values correspond to short position (i.e., borrowing).

10 This price index also satisfies an analogous expression for entrepreneurs’ consumption.
The household’s optimality conditions are given by:

\[
\Phi_L^\nu = \frac{W_t}{P_t}, \quad (2)
\]

\[
\frac{1 - \gamma}{\gamma} c_{H,t}^T = \frac{P_t^N}{P_t^T}, \quad (3)
\]

\[
\Lambda_{H,t} = \left( \frac{\psi_{H,t+1}}{\psi_{H,t}} \right) R_t E_t \left[ \Lambda_{H,t+1} \right], \quad (4)
\]

\[
\Lambda_{H,t} = \left( \frac{\psi_{H,t+1}}{\psi_{H,t}} \right) R_{t+1} E_t \left[ \frac{P_t^T}{P_{t+1}^T} \Lambda_{H,t+1} \right], \quad (5)
\]

\[
\Lambda_{H,t} = \chi \left( \frac{M_{H,t}}{P_t} \right)^{-1} + \left( \frac{\psi_{H,t+1}}{\psi_{H,t}} \right) E_t \left[ \frac{P_t^T}{P_{t+1}^T} \Lambda_{H,t+1} \right], \quad (6)
\]

where \( \Lambda_{H,t} \equiv \left( c_{H,t} - \Phi_{L,1+\nu} \right)^{-\sigma} \gamma \left( c_{H,t}^N / c_{H,t}^T \right)^{1-\gamma} \) is date \( t \) marginal utility of tradable goods consumption, and \( \psi_{H,t+1}/\psi_{H,t} \) is the discount factor between dates \( t \) and \( t + 1 \). Condition \( (2) \) describes optimal labor supply as a function of the real wage only – a consequence of the GHH preference specification. Equation \( (3) \) is a static condition linking the household’s marginal rate of substitution between the two consumption goods to their relative price. Equations \( (4) \) and \( (5) \) are Euler equations for foreign and local currency bond holdings. Finally, condition \( (6) \) represents the household’s Euler equation for holding money balances.

3.2. Entrepreneurs

Entrepreneurs’ expected life-time utility is given by:

\[
E_0 \sum_{t=0}^{\infty} \psi_{E,t} \left[ \frac{(c_{E,t})^{1-\sigma}}{1-\sigma} + \chi \ln \left( \frac{M_{E,t}}{P_t} \right) \right],
\]

where \( c_{E,t} \) is consumption, \( \frac{M_{E,t}}{P_t} \) is real money balances, and \( \psi_{E,t} \) is a discount factor. As for households, the consumption index \( c_{E,t} \) is defined as \( c_{E,t} \equiv \left( c_{E,t}^T \right)^{\gamma} \left( c_{E,t}^N \right)^{1-\gamma} \), where \( c_{E,t}^T \) and \( c_{E,t}^N \) denote the entrepreneur’s tradable and non-tradable goods consumption, respectively. \( \psi_{E,t} \) is an endogenous discount factor that depends on the aggregate (economy-wide) level of entrepreneurs’ consumption and is defined as \( \psi_{E,0} = 1 \) and \( \psi_{E,t} \equiv (\phi \beta)^{t} \prod_{k=0}^{t-1} c_{E,k}^{-\mu} \). \( \phi < 1 \) reflects the relative impatience of entrepreneurs vis-à-vis households and foreign investors, whose role is to generate equilibrium borrowing by entrepreneurs.

Entrepreneurs face a sequence of budget constraints given by:

\[
P_t^T c_{E,t}^T + P_t^N c_{E,t}^N + B_{E,t} + P_t^T f_{E,t} + M_{E,t} = R_t B_{E,t-1} + R_t^* P_t^T f_{E,t-1} + M_{E,t-1} + P_t^N A^N L_t^N - W_t L_t + P_t^T y_{E,t} + T_{E,t}, \quad (7)
\]

where \( y_{E,t}^T \) is the entrepreneur’s tradable goods endowment, \( B_{E,t} \) and \( f_{E,t} \) are, respectively, the entrepreneur’s holding of local and foreign currency bonds, and \( T_{E,t} \) is a lump-sum transfer. Each period, an entrepreneur receives a tradable goods endowment, revenues from
non-tradable output sale, as well as returns on past money and bond holdings. It chooses how to spend these proceeds between consumption, bond investment and money holding.

An entrepreneur’s optimality conditions for consumption and portfolio choices are given by:

\[
\frac{1 - \gamma}{\gamma} \frac{c_{E,t}^T}{C_{E,t}^N} = \frac{P_t^N}{P_t^T},
\]

(8)

\[
\Lambda_{E,t} = \left( \frac{\psi_{E,t+1}}{\psi_{E,t}} \right) R_t^* E_t \left[ \frac{P_{t+1}^T}{P_{t+1}^T} \Lambda_{E,t+1} \right],
\]

(9)

\[
\Lambda_{E,t} = \left( \frac{\psi_{E,t+1}}{\psi_{E,t}} \right) R_{t+1}^* E_t \left[ \frac{P_t^T}{P_{t+1}^T} \Lambda_{E,t+1} \right],
\]

(10)

\[
\Lambda_{E,t} = \chi \left( \frac{M_{E,t}}{P_t^T} \right) \frac{1}{P_t^T} + \left( \frac{\psi_{E,t+1}}{\psi_{E,t}} \right) E_t \left[ \frac{P_t^T}{P_{t+1}^T} \Lambda_{E,t+1} \right],
\]

(11)

where \( \Lambda_{E,t} \equiv c_{E,t}^{-\gamma} \left( c_{E,t}^N / c_{E,t}^T \right)^{1-\gamma} \) is date \( t \) marginal utility of tradable consumption, and \( \psi_{E,t+1} / \psi_{E,t} \) is the discount factor between dates \( t \) and \( t + 1 \). The entrepreneur’s optimality conditions for intra-temporal consumption allocation (8), foreign currency bond (9), local currency bond (10) and money balances (11) are analogous to the household’s optimality conditions (3), (4), (5) and (6).

Following standard practice in the New-Keynesian literature, we introduce nominal rigidities in the non-tradable goods market by (1) separating the sector into monopolistically competitive intermediate producers and perfectly competitive retailers, and (2) assuming Calvo (1983) price setting by intermediate producers. An entrepreneur is an intermediate producer \( j \) who produces a differentiated good \( y_{j,t} \) using labor \( L_{j,t} \) according to a linear production function \( y_{N,t} = A^N L_{j,t} \), where \( A^N \) is a (constant) productivity parameter. Entrepreneurs sell their output to competitive retailers who combine differentiated inputs to produce a final good using a constant elasticity of substitution (CES) production function

\[
Y_{N,t} = \left[ \int_0^1 (y_{j,t})^{a-1} \right]^{\frac{1}{a-1}},
\]

where \( a \) is the elasticity of substitution between any two differentiated intermediate non-tradable goods, and \( \mathcal{M} \equiv \frac{1}{1-a} \) is the entrepreneurs’ markup.

Following Calvo (1983), we assume that each period, only a fraction \( 1 - \theta \) of entrepreneurs can adjust their price. Accordingly, every entrepreneur faces a probability \( \theta \) of not being able to adjust its price in a given period. In this context, the entrepreneur’s optimal price setting strategy results in an aggregate non-tradable goods price dynamics which, at a first-order approximation, satisfies a standard New-Keynesian Phillips Curve:

\[
\pi_{t}^N = c_{E}^{-\mu} \phi \beta E_t \left[ \pi_{t+1}^N \right] + \lambda \left[ \log (MC_t) - \log (MC) \right]
\]

(12)

where \( \pi_{t}^N \equiv \log P_t^N - \log P_{t+1}^N \), \( c_{E}^{-\mu} \phi \beta \) is the entrepreneur’s steady state discount factor, \( \lambda = \frac{(1-\theta)(1-c_{E}^{-\mu} \phi \beta)}{\theta} \) is a composite parameter, \( MC_t \) is the entrepreneur’s real marginal cost (of which \( MC \) is the steady state value). In a given period, entrepreneurs who get a chance to re-set prices do as a function of future expected marginal costs. As a result, non-tradable
goods price inflation is a function of a discounted sum of expected future marginal costs. In the fully flexible price benchmark (i.e. $\theta = 0$), entrepreneurs set their price as a markup over marginal costs: $P_t^N = \mathcal{M} \times MC_t$, with $MC_t \equiv \frac{W_t}{\lambda}$.

3.3. Foreign lenders

Foreign lenders are assumed to be small and risk-averse. This is a crucial deviation from the literature and is meant to capture the fact that due to factors such as informational barriers, global investors only invest in a small number of foreign markets.\[11] A foreign lender’s expected life-time utility is given by:

$$E_0 \sum_{t=0}^{\infty} \psi_{F,t} \left( \frac{c_{F,t}}{c_{F}} \right)^{1-\sigma} \frac{1}{1-\sigma},$$

where $c_{F,t}$ is tradable goods consumption and $\psi_{F,t}$ is an endogenous discount factor that depends on the aggregate level of foreign lenders’ consumption and is defined as $\psi_{F,0} = 1$ and $\psi_{F,t} \equiv \beta^t \prod_{k=0}^{t-1} \left( \frac{c_{F,k}}{c_{F}} \right)^{-\mu}$ ($c_F$ is steady state consumption).

Foreign lenders face a sequence of budget constraints given by:

$$P_t^T c_{F,t} + B_{F,t} + P_t^T f_{F,t} = R_t B_{F,t-1} + R^* P_t^T f_{F,t-1} + y_{F,t},$$

(13)

where $B_{F,t}$ is the holding of local currency bonds, $f_{F,t}$ is the holding of foreign currency bonds, and $y_{F,t}$ is a tradable goods endowment. Each period, a foreign lender receives a stochastic endowment of the tradable good as well as returns on past bond holdings. It chooses how to spend these proceeds between consumption and bond investment.

The foreign lender’s optimality conditions are given by:

$$\Lambda_{F,t} = \left( \frac{\psi_{F,t+1}}{\psi_{F,t}} \right) R^* E_t \left[ \Lambda_{F,t+1} \right]$$

(14)

$$\Lambda_{F,t} = \left( \frac{\psi_{F,t+1}}{\psi_{F,t}} \right) R_{t+1} E_t \left[ \frac{P_t^T}{P_{t+1}^T} \Lambda_{F,t+1} \right]$$

(15)

where $\Lambda_{F,t} \equiv c_{F,t}^{\sigma}$ is date $t$ marginal utility of consumption, and $\psi_{F,t+1}/\psi_{F,t}$ is the discount factor between date $t$ and $t + 1$. The foreign lender’s optimality conditions for foreign currency bonds (14) and local currency bonds (15) are analogous to the household’s and entrepreneur’s Euler equations.

3.4. Government policy

We abstract from government expenditures and public debt accumulation. The domestic economy’s government therefore simply rebates revenues from money creation in a lump-sum

\[11\]This assumption is consistent with the empirical evidence reported by Didier et al. (2010), who find that U.S. equity mutual funds that operate globally only invest in a surprisingly limited number of stocks.
and directed fashion. Revenue raised with households is rebated to households and revenues raised with entrepreneurs is rebated to entrepreneurs:

\[ T_{H,t} = M_{H,t} - M_{H,t-1}, \]
\[ T_{E,t} = M_{E,t} - M_{E,t-1}. \]

### 3.5. Fundamentals

The model economy features two sources of uncertainty: real shocks and nominal shocks. Real shocks cause fluctuations in tradable goods endowments of the various agents, while nominal shocks lead to fluctuations in the domestic money supply.\(^\text{12}\) Shocks follow the following AR(1) processes:

\[
\begin{align*}
\ln(z_t) &= \rho_T \ln(z_{t-1}) + \epsilon_{T,t}, \\
\ln(y_{F,t}) &= \rho_F \ln(y_{F,t-1}) + \epsilon_{F,t}, \\
\ln(M_t) &= \rho_M \ln(M_{t-1}) + \epsilon_{M,t}.
\end{align*}
\]

where \(\epsilon_{T,t}, \epsilon_{F,t}, \epsilon_{M,t}\) are mean zero, normally distributed, disturbances that are i.i.d. over time with respective variances \(\sigma^2_T, \sigma^2_F, \sigma^2_M\), and \(\rho_T, \rho_F, \rho_M\) are autoregressive coefficients. \(z_t\) is the sole real driver of fluctuations in the domestic economy, governing the tradable goods endowments of both households and entrepreneurs according to

\[ y_{T,H,t} = Y_H z_t \]
\[ y_{T,E,t} = Y_E z_t \]

for some scale parameters \(Y_H, Y_E\). \(y_{F,t}\) represents real fluctuations in the world economy and can be interpreted as a measure of the world business cycle. The correlation between domestic endowment shocks \(z_t\) and foreign endowment shocks \(y_{F,t}\) is given by \(\rho_{T,F} > 0\), while monetary shocks are assumed to be uncorrelated to real shocks.

### 3.6. Equilibrium

The model features a number of markets which need to clear in equilibrium. On the market for the final non-tradable goods, demand from households and entrepreneurs needs to equate supply from retailers:

\[ c_{N,H,t} + c_{N,E,t} = A^N L_t. \]

On the market for domestic money, demand by households and entrepreneurs has to coincide with the (exogenous) money supply:

\[ M_{H,t} + M_{E,t} = M_t. \]

The price of the tradable good \(P^T_t\), which coincides with the nominal exchange rate \(E_t\), thus adjusts to ensure that in equilibrium the demand for real balances by households and entrepreneurs matches the real value of the currency issued by the central bank. We elaborate more on the determination of this exchange rate in Section \(\text{3.7}\). The market for local currency

\(^{12}\text{For simplicity, we abstract from stochastic productivity fluctuations in the non-tradable goods sector.}\)
bonds features zero net supply:
\[ B_{H,t} + B_{E,t} + B_{F,t} = 0. \quad (21) \]

Finally, demand for tradable goods consumption by domestic households and entrepreneurs has to match domestic tradable good production and imports from abroad (financed via current account imbalances). Defining the entrepreneur’s and household’s net worth (in terms of the tradable good) as \( a_{E,t} \equiv f_{E,t} + b_{E,t} \) and \( a_{H,t} \equiv f_{H,t} + b_{H,t} \) (for \( b_{i,t} \equiv B_{i,t}/P^T_t \)), tradable goods market clearing requires:
\[
\begin{align*}
\kappa^T_{E,t} + \kappa^T_{H,t} + a_{E,t} + a_{H,t} &= R^*(f_{E,t-1} + f_{H,t-1}) + \left( R_t \frac{P^T_{t-1}}{P^T_t} \right)(b_{E,t-1} + b_{H,t-1}) + y^T_{H,t} + y^T_{E,t}. 
\end{align*}
\quad (22)
\]

An equilibrium is a set of sequences for allocations, portfolio choices and prices such that all agents optimize and markets clear.

3.7 Forces at play

Equilibrium portfolio choices and risk premia result from the interplay between two elements: (1) the risk profiles of local and foreign currency bonds, as well as (2) the exposure of the various agents to these risks. We now discuss these two elements in turn.

In terms of the tradable consumption good, the foreign currency bond is riskless. The local currency bond, on the other hand, is risky as its payoff depends on nominal exchange rate fluctuations. Its rate of return is low when the local currency depreciates and high when it appreciates. A local currency depreciation, in turn, may arise as the result of negative shocks to domestic tradable endowments or positive shocks to the domestic money supply. Negative shocks to the domestic tradable endowments lower the income of domestic agents and thus reduce their demand for consumption and real balances. For a given nominal money supply, the reduction in real balances is brought about by a local currency depreciation. Positive shocks to the domestic money supply also lead to a local currency depreciation because a reduction in the purchasing power of money is required for a larger money supply to match with a stable demand for real balances by domestic agents.\(^\text{13}\)

The relative willingness of domestic households and foreign lenders to hold local currency bonds naturally depends on the sign and magnitude of the covariance between their stochastic discount factor (SDF) and the return on these bonds. Their SDFs are respectively given by
\[
\begin{align*}
\mathbb{M}^H_{t,t+1} &= \beta_c c^{-\mu}_H \left( \frac{\Lambda_{H,t+1}}{\Lambda_{H,t}} \right) \\
\mathbb{M}^F_{t,t+1} &= \beta \left( \frac{c_{F,t}}{c_{F,t}} \right)^{-\mu} \left( \frac{\Lambda_{F,t+1}}{\Lambda_{F,t}} \right),
\end{align*}
\]
and the equilibrium risk premium

\(^{13}\)Similarly, a local currency appreciation may arise as the result of positive shocks to domestic tradable endowments or negative shocks to the domestic money supply.
on local currency bonds is related to above mentionned covariance according to:

\[
E_t \left[ \frac{R_{t+1}P_{t+1}^D}{P_{t+1}^T} \right] - R^* = - \frac{\text{Cov}_t \left( \frac{M_{t,t+1}^H}{P_{t+1}^T}, \frac{R_{t+1}P_{t+1}^D}{P_{t+1}^T} \right)}{E_t[M_{t,t+1}^H]},
\]

\[
E_t \left[ \frac{R_{t+1}P_{t+1}^D}{P_{t+1}^T} \right] - R^* = - \frac{\text{Cov}_t \left( \frac{M_{t,t+1}^F}{P_{t+1}^T}, \frac{R_{t+1}P_{t+1}^D}{P_{t+1}^T} \right)}{E_t[M_{t,t+1}^F]}.
\]

The agents’ SDF increases in future marginal utility and thus decreases in future tradable consumption.

Under fully flexible prices, money is neutral and nominal shocks are therefore uncorrelated with the domestic households’ and foreign lenders’ SDFs. But tradable productivity shocks make the return on local currency bonds positively correlated with the domestic households’ tradable consumption, and thus negatively correlated with their SDF. This is because following a positive tradable productivity shock, tradable consumption increases while the local currency appreciates. As a result, for low levels of correlation between the domestic economy’s tradable productivity and the foreign lenders’ endowments, holding the local currency bond is a worse hedge for domestic households than for foreign lenders. This suggests that under flexible prices, foreign lenders demand a lower risk premium than domestic households on local currency bonds, and are therefore natural holders of these bonds.

The picture changes significantly when sticky prices come into play. Under nominal rigidities in the non-tradable sector, nominal shocks are not neutral any more. In particular, positive (negative) money supply shocks are expansionary (contractionary) due to their effect on aggregate demand. These shocks contribute to making returns on local currency bonds negatively correlated with domestic households’ tradable consumption, and thus positively correlated with their SDF. Larger degrees of price stickiness magnify this effect, which acts counter to the effect of real shocks discussed above. As a result, for larger degrees of price stickiness, holding the local currency bond turns out to be a better hedge for domestic households. This suggests that for a larger degree of nominal rigidities in the non-tradable sector, domestic households demand a smaller risk premium on local currency bonds, and may therefore deprive foreign lenders from their status of natural holders of these bonds.

4. Solution and calibration

4.1. Solving for portfolio choices

It is well known that solving for portfolio choices in open-economy macroeconomic models with incomplete markets is challenging\(^{14}\). Recent methods proposed by Devereux and Sutherland (2010), Tille and van Wincoop (2010), and Evans and Hnatkovska (2012) overcome the traditional challenges by making use of Samuelson (1970)’s principle that in order

\(^{14}\)Global methods are often impractical, due to the dimensionality of the problems involved. Standard perturbation methods, on the other hand, are inappropriate, since they approximate the model solution around the deterministic steady state, in which portfolio choices are not uniquely defined.
to derive a solution for portfolio choices up to $N$-th order accuracy, the portfolio problem must be approximated up to the $N+2$th order. We solve our model by extending these methods to accommodate situations with more than two agents. Details are presented in the Appendix.

4.2. Calibrations

We calibrate the model to annual data. Several parameter values are standard. The discount factor is set to $\beta = 0.96$. The coefficient of relative risk aversion (or inverse elasticity of intertemporal substitution) is set to $\sigma = 2$. The share of tradable consumption in total consumption is set to $\gamma = 0.3$, a value commonly used in small open economy models. The coefficient representing the weight of real balances in the domestic agents’ utility function is set to a low value of $\chi = 0.001$, with the aim to approximate a cash-less economy. The steady-state money supply is normalized to $M = 1$. The inverse Frisch elasticity of labor supply is set to $\nu = 0.6$, as in Greenwood et al. (1988). The elasticity of substitution between differentiated non-tradable goods is set to $\varepsilon = 10$, a common value in the New-Keynesian literature (see, for instance, Chugh 2006). The elasticity parameter of the endogenous discount factor is set to $\mu = 0.001$, so as to keep its impact on the equilibrium dynamics small (see Nguyen 2011). The steady state level of foreign output $y_F$ is immaterial for our results and is therefore normalized to unity. The Calvo price adjustment parameter $\theta$ is set to 0.25 so as to match an average frequency of price adjustment of 16 months, in line with the empirical evidence on services (excluding sales) documented by Nakamura and Steinsson (2008) using U.S. product-level price data.

Other parameters are jointly calibrated to target steady state quantities. The weight on the disutility of labor is set to $\Phi = 4.5$ so that steady-state employment is about one third. The non-tradable goods productivity is set to $A_N = 3.5$ to achieve a relative share of tradable output in total output of about one third. The relative impatience parameter and the tradable endowment levels are respectively set to $\phi = 0.999$, $Y_H = 0.57$ and $Y_E = 1.3$, to jointly target a steady state credit of domestic households to domestic entrepreneurs of 58% of GDP, and a net foreign asset position of 50% of GDP. These figures are close to the relevant numbers for Peru (1990-2012), a stable small open emerging economy.

Finally, the parameters pertaining to the shock processes are calibrated using annual data for Peru and the U.S. (taken as the foreign investor) from the World Development Indicators. From Peru’s money stock data we obtain $\rho_M = 0.55$ and $\sigma_M = 0.09$. From Peru’s HP-filtered GDP per capita data, we obtain $\rho_T = 0.36$ and $\sigma_T = 0.024$. From U.S.’s HP-filtered GDP per capita data, we obtain $\rho_F = 0.39$ and $\sigma_F = 0.010$. For the correlation measure, we obtain $\rho_{T,F} = 0.18$, but we also experiment with a wider range of values.

5. Relative shock exposures: intuition from exogenous liability dollarization

In order to build intuition about how the degree of price rigidity shapes the endogenous currency composition of liabilities, we start by considering a special case of the model in which debt is (exogenously) only denominated in foreign currency. In particular, we emphasize how absent local currency debt, agents’ consumption risk with respect to nominal and real
shocks hitting the domestic economy depends on the degree of price stickiness. We focus on the exposure of domestic households and entrepreneurs, since under exogenous liability dollarization foreign investors are not affected by these shocks.

We begin by considering the monetary shock. Figure 1 shows impulse responses of several key variables to a 1% increase in the money supply. Solid lines represent the flexible price case ($\theta = 0$), while dashed lines represent the limit of fully rigid prices ($\theta = 1$). Under flexible prices, money is neutral: real variables do not respond and nominal prices all adjust by the same amount. Thus, the local currency depreciates on impact, but the agents’ SDF does not move since none of the real variables react to the shock. Therefore, with flexible prices local currency bonds would neither be a good nor a bad hedge for both lenders and borrowers in response to monetary shocks. In contrast, under rigid non-tradable goods prices, money is not neutral anymore. A positive money supply shock is expansionary through a standard New-Keynesian channel. The decrease in the interest rate accompanying the increase in money supply stimulates aggregate demand, and given rigid prices in the non-tradable goods sector, this results in temporarily higher employment and output. Consumption of both households and entrepreneurs rises on impact, but the rise is much less pronounced for entrepreneurs, who see their markup decline in response to the shock (since nominal wages rise but non-tradable goods prices stay fixed). In this case, a long position in local currency bonds would provide a relatively better hedge for households than for entrepreneurs with respect to monetary shocks. The issuance of local currency debt by entrepreneurs to households would thus improve domestic nominal risk sharing. This illustrates that when prices are sticky, monetary shocks tend to make households natural holders of local currency bonds.

| $\beta$ | Discount factor | 0.96 |
| $\phi$ | Entrepreneurs’ relative impatience coefficient | 0.999 |
| $\mu$ | Endogenous discount factor coefficient | 0.001 |
| $\gamma$ | Share of tradable consumption | 0.3 |
| $\chi$ | Weight on real balances in utility | 0.001 |
| $\nu$ | Inverse Frish elasticity of labor supply | 0.6 |
| $\Phi$ | Weight on disutility of labor | 4.5 |
| $\varepsilon$ | Elasticity of input substitution between differentiated goods | 10 |
| $A_N$ | Non-tradable productivity | 3.5 |
| $Y_H$ | Household’s steady-state tradable endowment | 0.57 |
| $Y_E$ | Entrepreneur’s steady-state tradable endowment | 1.3 |
| $M$ | Long run money supply | 1 |
| $y_F$ | Foreign lender’s steady state income | 1 |
| $\rho_T$ | Persistence of the domestic tradable endowment shocks | 0.36 |
| $\rho_F$ | Persistence of foreign income shocks | 0.39 |
| $\rho_M$ | Persistence of the monetary shocks | 0.55 |
| $\sigma_T$ | Standard deviation of the domestic tradable endowment shocks | 0.024 |
| $\sigma_F$ | Standard deviation of foreign income shocks | 0.010 |
| $\sigma_M$ | Standard deviation of monetary shocks | 0.09 |

Table 1: Values of benchmark parameters
Figure 1: Impulse responses to a 1% increase to money supply.

Figure 2: Impulse responses to a 1% decrease to tradable endowment.
Next, we consider the real shock. Figure 2 shows impulse responses of the same key variables to a 1% decrease in the domestic tradable endowment. Again, solid lines represent the flexible price case ($\theta = 0$), while dashed lines represent the limit of fully rigid prices ($\theta = 1$). We begin by noting that all responses are extremely persistent, as the temporary endowment shock is a shock to the lifetime income of domestic agents. Under flexible prices, consumption decreases by a similar amount for households and entrepreneurs. The adverse tradable endowment shock lowers the demand for non-tradable goods and for money. As a result, employment drops, and nominal prices increase (the relative price of money balances drops). The nominal wage and non-tradable goods price increase less than the tradable goods price, resulting in a decrease in the real wage and relative price of non-tradable goods. Thus, consumption falls at the same time that the local currency depreciates. With flexible prices, a long position in local currency bonds would therefore be a bad hedge for domestic agents in response to real shocks. In contrast, under rigid prices, the consumption response is less homogenous. Households’ consumption rises, while entrepreneurs’ consumption falls on impact. Since non-tradable goods prices are rigid, their relative price drops sufficiently to stimulate demand and generate a rise in employment. Households thus receive higher wage income, and their consumption rises, while entrepreneurs see their mark-up erode and experience a consumption drop. In this case, a long position in local currency bonds would be a good hedge for households, but a bad hedge for entrepreneurs. This illustrates that when prices are sticky, real shocks also tend to make households natural holders of local currency bonds, and entrepreneurs natural issuers thereof.

6. Endogenous currency composition of liabilities

We now analyze the endogenous currency composition of liabilities when households, entrepreneurs and foreign investors have access to markets for foreign and local currency bonds. We do so by conducting two experiments. In the first one, we fix the degree of business cycle synchronization between the emerging economy and the rest of the world (i.e. the correlation between the domestic tradable endowment shock and the foreign lenders’ endowment shock), and compute the long-run currency composition of liabilities for varying degrees of nominal rigidity. In the second one, we fix the degree of nominal rigidity and compute the long-run currency composition of liabilities for varying degrees of business cycle synchronization.

In Figure 3, we fix the degree of business cycle synchronization to our calibrated value of $\rho_{T,F} = 0.18$, and represent the long-run level (technically, zero-th order) of local (top panels) and foreign (bottom panels) currency lending by foreigners (left panels) and by households (right panels) for varying degrees of nominal rigidity $\theta$. Lending amounts are normalized by the GDP of the emerging economy. Dotted vertical lines represent our calibrated value of $\theta = 0.25$. Several results stand out. First, while foreigners do nearly 60% of their lending to the domestic economy in local currency under flexible prices, foreign currency accounts for 15% of output in local currency, and about 20% of output in foreign currency.
Figure 3: Local currency lending by foreign lenders (left) and households (right) for mild business cycle synchronization ($\rho_{T,F} = 0.18$).

about 80% of their lending under sticky prices with a realistic degree of nominal rigidities corresponding to our calibrated value of $\theta = 0.25$. Second, while domestic households borrow in local currency under flexible prices, they turn to being net lenders in local currency for very mild degrees of nominal rigidities, starting at $\theta = 0.13$. At our calibrated value of $\theta = 0.25$, they even do more than 70% of their lending in local currency. Third, more severe degrees of nominal rigidities (i.e., above $\theta = 0.39$) would result in foreign lenders being pushed out of the local currency lending market.\textsuperscript{16}

\textsuperscript{16}In other words, the model predicts that for values of $\theta$ above 0.39, foreigners are net borrowers in local currency. This occurs because in the model there are no frictions that prevent short selling of bonds in local currency by foreigners. If such frictions were present, the model would instead predict an absence of market participation.
Our results thus indicate that a mild degree of nominal rigidity suffices to make the consumption basket effect potent enough to significantly change the profile of participants in the local currency bond market.

We now proceed to examining local currency lending from a different angle. In Figure 4, we consider the cases of flexible prices ($\theta = 0$) and sticky prices (under our retained calibration of $\theta = 0.25$), and represent the long-run equilibrium portfolios for varying degrees of business cycle synchronization $\rho_{T,F}$. Our calibrated value of $\rho_{T,F} = 0.18$ is represented by a dotted vertical line in all panels. Similarly to Figure 3, top panels show local currency lending and bottom panels show foreign currency lending, while left panels show lending by foreigners and right panels show lending by domestic households.

It is apparent from the figure that the degree of business cycle synchronization matters importantly for the foreign lenders’ currency position, but only marginally for that of domestic households. Under both flexible and sticky prices, the foreign lenders’ long position
in local currency bonds decreases with the degree of business cycle synchronization, while its long position in foreign currency increases with this measure. Since the local currency depreciates when the emerging economy’s tradable endowment is low, the larger the correlation between the emerging economy’s tradable endowment and the foreign lenders’ endowment, the less attractive are local currency bonds for foreign lenders from a hedging perspective. In a benchmark with no business cycle synchronization (i.e., a correlation of $\rho_{T,F} = 0$), foreign lenders do 90% of their lending to the emerging economy in local currency when prices are flexible, compared with about 50% when prices are sticky. In contrast, under our $\rho_{T,F} = 0.18$ calibration, foreign lenders do 60% of their lending to the emerging economy in local currency when prices are flexible, compared with a mere 20% when prices are sticky. The model predicts that for higher degrees of business cycle synchronization, foreign lenders eventually turn into issuers of local currency bonds. The threshold above which this seemingly unrealistic phenomenon occurs is significantly lower under sticky prices ($\rho_{T,F} = 0.33$) than under flexible prices ($\rho_{T,F} = 0.58$).

7. Conclusion

This paper provides a new explanation of why developing countries may not borrow much internationally in their local currency. We show that in the presence of nominal rigidities in the non-tradable goods sector, the hedging properties of local currency bonds shift in favor of domestic residents holding such bonds. This occurs because in that case nominal shocks contribute to generating a positive correlation between the return on local currency bonds and domestic agents’ stochastic discount factor. This mechanism can thus explain a limited participation of foreign lenders in emerging economies’ local currency bond markets. We present a simple DSGE model to provide a quantitative assessment of the mechanism. For plausible parameter values, the analysis suggests that differences in consumption baskets coupled with nominal rigidities can go a long way in explaining external liability dollarization, without having to resort to financial market frictions or exchange rate manipulation arguments.

Bibliography


Bordo, M. D., Meissner, C., Redish, A., 2005. How original sin was overcome: The evolution of external debt denominated in domestic currencies in the united states and the

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17 Once again, this happens because the model does not feature frictions that would prevent short-selling of local currency bonds by foreigners. If such frictions were present, the model would instead predict no participation by foreigners in the local currency bond market.
british dominions. In: Eichengreen, B., Hausmann, R. (Eds.), Other people’s money: debt denomination and financial instability in emerging market economies. The University of Chicago Press.


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Appendix: solving for portfolio choices

This appendix presents the solution for the zero-order portfolio choices in the case where households and entrepreneurs trade both foreign currency bonds and local currency bonds with foreign investors, and where we make the assumption that the foreign lenders are small and risk averse. Define the entrepreneurs’ and households’ real holdings of local currency bonds as \( b_{E,t} \equiv B_{E,t}/P_t^T \), \( b_{H,t} \equiv B_{H,t}/P_t^T \) and \( b_{F,t} \equiv B_{F,t}/P_t^T \) and their net worth as \( a_{E,t} \equiv f_{E,t} + b_{E,t} \), \( a_{H,t} \equiv f_{H,t} + b_{H,t} \) and \( a_{F,t} \equiv f_{F,t} + b_{F,t} \). The only equilibrium conditions of the model where these local currency bond and net worth positions show up are the entrepreneurs’
budget constraint, the foreign lenders’ budget constraint and and the economy’s resource
constraint. We rewrite the equations:
\[ c_{E,t}^T + p_t^N c_{E,t}^N + a_{E,t} = R_t^* a_{E,t-1} + (R_t \frac{P_{t-1}^T}{P_t^T} - R_t^*) b_{E,t-1} + p_t^N A^N L_t^N \]
\[ + y_{E,t} (L_t^N + L_t^T). \] (A.1)

and
\[ c_{F,t} + a_{F,t} = R_t^* a_{F,t-1} + (R_t \frac{P_{t-1}^T}{P_t^T} - R_t^*) b_{F,t-1} + y_{F,t} \] (A.2)

and
\[ c_{E,t} + c_{H,t} + a_{E,t} + a_{H,t} = R_t^* (a_{E,t-1} + a_{H,t-1}) + \left( \frac{R_t P_{t-1}^T}{P_t^T} - R_t^* \right) (b_{E,t-1} + b_{H,t-1}) + y_{E,t} + y_{H,t} \] (A.3)

Denote the first-order components of the excess return of the portfolio of entrepreneurs and households as $\epsilon_t^E \equiv b_E R_t (\tilde{P}_{t-1}^T - \hat{P}_t^T + \tilde{R}_t - \hat{R}_t^*)$ and $\epsilon_t^H \equiv b_H R_t (\tilde{P}_{t-1}^T - \hat{P}_t^T + \tilde{R}_t - \hat{R}_t^*)$. Following the approach of Devereux and Sutherland, we initially consider $\epsilon_t^E$ and $\epsilon_t^H$ to be exogenous i.i.d. random variables. The first-order approximations of the terms $(R_t \frac{P_{t-1}^T}{P_t^T} - R_t^*) b_{E,t-1}$ in (A.1), $(R_t \frac{P_{t-1}^T}{P_t^T} - R_t^*) b_{F,t-1}$ in (A.2) and $(R_t \frac{P_{t-1}^T}{P_t^T} - R_t^*) (b_{E,t-1} + b_{H,t-1})$ in (A.3) are expressed by $\epsilon_t^E$, $-(\epsilon_t^E + \epsilon_t^H)$ and $\epsilon_t^E + \epsilon_t^H$ respectively.

From this we can solve for the first order approximation of the model, with $\epsilon_t^E$ and $\epsilon_t^H$ as the two iid state variables. Rearranging terms gives us the first-order accurate solution for the excess return on local currency bonds:
\[ \hat{r}_{x,t+1} = \theta_r \epsilon_{t+1}^H + \theta_r^E \epsilon_{t+1}^E + \theta_r \epsilon_{t+1} \] (A.4)

where $\epsilon_{t+1}$ is the vector of the exogenous shocks. Utility differences between foreign lenders and entrepreneurs, and households and entrepreneurs, respectively are:
\[ (co_{cht} \hat{c}_{H,t+1}^T + co_{chh} \hat{c}_{H,t+1}^T + co_{ihat} - co_{ceu} \hat{c}_{E,t+1} - co_{ceu} \hat{c}_{E,t+1}) \]
\[ = \theta_{H,E} \epsilon_{t+1}^H + \theta_{H,E} \epsilon_{t+1}^E + \theta_{F,E} \epsilon_{t+1}^E + \theta_{F,E} \epsilon_{t+1}^E + \theta_{F,E} \epsilon_{t+1} + \hat{\theta}_{F,E} \hat{x}_t \] (A.5)
\[ (\sigma \hat{c}_{F,t+1} + (\gamma - 1 - \sigma \gamma) \hat{c}_{E,t+1} + (1 - \gamma)(1 - \sigma) \hat{c}_{E,t+1}) \]
\[ = \theta_{H,E} \epsilon_{t+1}^H + \theta_{H,E} \epsilon_{t+1}^E + \theta_{H,E} \epsilon_{t+1} + \hat{\theta}_{H,E} \hat{x}_t \] (A.6)

where $\hat{x}_t$ is the vector of the endogenous state variables.

Recognizing that $\epsilon_{t+1}^H = b_H R \hat{r}_{x,t+1}$ and $\epsilon_{t+1}^E = b_E R \hat{r}_{x,t+1}$, we substitute them back into (A.4) and derive:
\[ \hat{r}_{x,t+1} = \frac{\theta_r \epsilon_{t+1}}{1 - \theta_r^H b_H R - \theta_r^E b_E R} \] (A.7)

---

18 The households’ budget constraint can be obtained by combining these constraints.
and into (A.5), (A.6) to obtain:

\[
(co_{cht}c_{H,t+1}^T + co_{chn}c_{H,t+1}^T + co_l - co_{ceh}c_{E,t+1}^T - co_{cne}c_{E,t+1}^N)
= (\theta_{F,E}^H b_H R + \theta_{F,E}^E b_E R) \frac{\theta_r \varepsilon_{t+1}}{1 - \theta_r^H b_H R - \theta_r^E b_E R} + \theta_{F,E} \varepsilon_{t+1} + \tilde{\theta}_{F,E} \hat{x}_t \tag{A.8}
\]

\[
(\sigma \hat{\epsilon}_{F,t+1} + (\gamma - 1 - \sigma \gamma)\hat{c}_{E,t+1}^T + (1 - \gamma)(1 - \sigma)\hat{c}_{E,t+1}^N)
= (\theta_{H,E}^H b_H R + \theta_{H,E}^E b_E R) \frac{\theta_r \varepsilon_{t+1}}{1 - \theta_r^H b_H R - \theta_r^E b_E R} + \theta_{H,E} \varepsilon_{t+1} + \tilde{\theta}_{H,E} \hat{x}_t \tag{A.9}
\]

Next, we substitute (A.8) and (A.9) into second-order approximations of the Euler equations of households, entrepreneurs and foreign lenders:

\[
E_t[(\hat{\epsilon}_{t+1}^B - \hat{\epsilon}^*) (co_{cht}c_{H,t+1}^T + co_{chn}c_{H,t+1}^T + co_l - co_{ceh}c_{E,t+1}^T - co_{cne}c_{E,t+1}^N)] = 0, \tag{A.10}
\]

\[
E_t[(\hat{\epsilon}_{t+1}^B - \hat{\epsilon}^*) (\sigma \hat{\epsilon}_{F,t+1} + (\gamma - 1 - \sigma \gamma)\hat{c}_{E,t+1}^T + (1 - \gamma)(1 - \sigma)\hat{c}_{E,t+1}^N)] = 0, \tag{A.11}
\]

After substituting (A.8) and (A.9) into (A.10) and (A.11), they become:

\[
\begin{align*}
[Rb_H(\theta_{F,E}^H \theta_r - \theta_{F,E}^H \theta_{F,E}) + Rb_E(\theta_{F,E}^E \theta_r - \theta_{F,E}^E \theta_{F,E}) + \theta_{F,E}] \Sigma \theta'_r &= 0 \tag{A.12} \\
[Rb_H(\theta_{H,E}^H \theta_r - \theta_{H,E}^H \theta_{H,E}) + Rb_E(\theta_{H,E}^E \theta_r - \theta_{H,E}^E \theta_{H,E}) + \theta_{H,E}] \Sigma \theta'_r &= 0 \tag{A.13}
\end{align*}
\]

where \( \Sigma = E_t \varepsilon_{t+1} \varepsilon'_{t+1} \).

From the two equations above, we can derive \( b_E \) and \( b_F \) as follows: Denoting \( a_1 \equiv R(\theta_{F,E}^H \theta_r - \theta_{F,E}^H \theta_{F,E}) \Sigma \theta'_r, b_2 \equiv R(\theta_{F,E}^E \theta_r - \theta_{F,E}^E \theta_{F,E}) \Sigma \theta'_r, a_3 \equiv \theta_{F,E} \Sigma \theta'_r, \) and \( d_1 \equiv R(\theta_{H,E}^H \theta_r - \theta_{H,E}^H \theta_{H,E}) \Sigma \theta'_r, d_2 \equiv R(\theta_{H,E}^E \theta_r - \theta_{H,E}^E \theta_{H,E}) \Sigma \theta'_r, d_3 \equiv \theta_{H,E} \Sigma \theta'_r \), (A.12) and (A.13) become:

\[
\begin{align*}
a_1 b_H + a_2 b_E + a_3 &= 0, \\
d_1 b_H + d_2 b_E + d_3 &= 0.
\end{align*}
\]

Hence,

\[
b_E = \frac{a_1 d_3 - a_3 d_1}{a_2 d_1 - a_1 d_2} \quad \text{and} \quad b_H = \frac{a_2 d_3 - a_3 d_2}{a_1 d_2 - a_2 d_1},
\]

and \( b_F = -(b_E + b_H) \).