Macro-Prudential Policy Coordination

Julien Bengui *

Université de Montréal

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Abstract

In the presence of financial market imperfections, pecuniary externalities may provide a rationale for macro-prudential liquidity regulation. Using a multi-country model of liquidity demand, we argue that they necessarily also call for an *international coordination* of such policy interventions. Absent coordination, national regulators fail to internalize the share of externalities operating across borders and are tempted to manipulate the price of liquidity during crises to shift surplus from foreign to domestic agents. As a consequence of these state-contingent terms-of-trade manipulations, uncoordinated liquidity regulation is generally inferior to coordinated regulation and may even be dominated by the laissez-faire.

Keywords: Systemic crises, international coordination, macro-prudential regulation, interbank markets

JEL Codes: F36, G21, F42, E44, D62

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Corresponding e-mail: `julien.bengui@umontreal.ca`

1 Introduction

The harmonization of financial regulation across national jurisdictions has been a key theme in banking for nearly three decades. For most of this period, however, the debate focussed mainly on the reasons for and consequences of leveling the playing field in the area of capital requirements. It is only following the outbreak of the global financial crisis in 2007 that the international coordination of liquidity standards reached the top of policymakers' agenda in the context of the Third Basel Accord (Basel III). Proponents of such a coordination of liquidity standards argue that it would avoid a regulatory race-to-the-bottom in which countries free-ride on the soundness and resilience of foreign banking systems. This paper formally examines this argument. We view an explicit reference to systemic risk as the key distinction between the recent discussion on liquidity regulation and the traditional approach to capital regulation. Accordingly, we model the welfare theoretic motivation for liquidity regulation as arising from an externality operating on the market for liquidity during financial crises. By limiting agents' exposure to liquidity risk ex-ante, regulators can reduce the scarcity of liquidity during crises and thereby correct a market failure caused by a pecuniary externality under incomplete markets. But when the market for liquidity has a global scale, domestically-minded national regulators fail to adequately internalize the share of the externality that operates across borders. A failure to coordinate liquidity standards across countries therefore generally results in regulations that are too lax from a social point of view, leading to excessively severe and costly financial crises ex-post.

The analysis is undertaken in the context of a two-country version of a model of liquidity demand.[1] Ex-ante identical agents invest in risky long-term projects that may require an additional liquidity injection along the road. Liquidity shocks are imperfectly correlated across countries, implying opportunities for international risk-sharing. Cross-country insurance against these shocks is limited, but agents can set aside liquid resources ex-ante by investing in a short-term asset (i.e. self-insure), or alternatively, they can borrow ex-post on an international spot market up to some limit. In this environment, market incompleteness results in a constrained inefficiency of the competitive equilibrium. Agents fail to internalize that

[1] For analytical convenience, we use a model of firm liquidity demand in the spirit of Holmstrom and Tirole (1998) rather than one of consumer liquidity demand à la Diamond and Dybvig (1983).

their collective investment choice affects the severity of a potential credit crunch, and they underinvest in short-term assets in equilibrium. Curbing agents' exposure to liquidity risk via prudential regulation can restore constrained efficiency. We compare three alternative allocation mechanisms: (i) the laissez-faire outcome (competitive equilibrium), (ii) the constrained efficient outcome achieved by setting liquidity regulation cooperatively at a global level, and (iii) the equilibrium of a policy game where regulation is chosen non-cooperatively by welfare maximizing national authorities.

The inefficiency of decentralized investment decisions in the model results from a pecuniary externality operating via the international interest rate in a crisis. This interest rate depends on the scarcity of global liquidity. At the margin, a rebalancing of the representative agent's investment portfolio from its laissez-faire value in favor of the short-term asset lowers the interest rate and causes a redistribution of wealth from lenders to borrowers in a crisis. Due to the incompleteness of markets, borrowers value liquidity more highly than lenders ex-post. Since ex-ante, any agent could end up with a high or with a low valuation of liquidity, a marginally lower interest rate during crises achieves a redistribution of resources from low valuation states of nature to high valuation states. Such a redistribution partially substitutes for missing risk markets and leads to a first order welfare gain. A global planner would have agents tilt their investment in favor of short-term assets, with the consequence of alleviating credit crunches when a crisis occurs.

National planners, who set regulation non-cooperatively to maximize the welfare of a domestic representative agent, do perceive the dependence of the severity of a potential credit crunch upon ex-ante investment choices. But in contrast to a global planner, they rationally attempt to shift surplus in favor of residents rather than restore constrained efficiency. In particular, national planners recognize that more domestic ex-ante investment in short-term assets would contribute to alleviate a foreign credit crunch in states of nature where foreigners with a high valuation of liquidity borrow from domestic lenders with a low valuation of liquidity. But national planners derive no benefits from alleviating credit crunches abroad. Since the pecuniary externality operates across borders, national planners do not internalize the full exchange efficiency benefits of regulation. Consequently, national planners generally fall short of imposing the optimal extent of liquidity regulation. This underprovision of ex-ante regulation results in more severe and more costly financial crises ex-post, in the form of larger interest rate spikes and more forced liquidation of real investments. In fact, national planners' incentives to manipulate state-contingent dynamic terms-of-trade (i.e. state-contingent interest rates) in favor of residents can cause the equilibrium of the non-cooperative regulation game to feature even less liquid and more risky investments than the laissez-faire benchmark. When this occurs, welfare is lower with uncoordinated liquidity regulation than under the laissez-faire. In other words, uncoordinated regulation can be worse than no regulation at all.

The constrained inefficiency of competitive equilibria in incomplete markets economies is well known since the work of Hart (1975), Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). So too is the suboptimality of uncoordinated macroeconomic policies since at least Johnson (1965) and Hamada (1976). The novelty of the present paper is to analyze in a common framework the interplay between distortions arising from market incompleteness and those resulting from openness and countries' monopoly and monopsony power in global markets. The analysis outlines the close link between the mechanics of policy incentives arising from these two kinds of distortions. A constrained global planner internalizes pecuniary externalities in much the same way as strategically acting governments do. But the former does so to improve exchange efficiency and reduce the cross-sectional wedges between marginal rates of substitutions caused by incomplete markets. In contrast, the latter use market power to shift surplus in their favor, possibly at the cost of widening these wedges and reducing overall efficiency.

Besides its implications for policy coordination, the model offers predictions about the international spillover effects of changes in liquidity regulation. A regulation-induced decrease in liquidity risk-taking in one country increases liquidity risk-taking in the other country. The transmission channels work through the lowering of interest rates during potential future crises brought about by the extra amount of liquidity set aside in the regulated country. The model also predicts that macro-prudential policies are strategic substitutes across countries, as a country's tightening of regulation, by increasing the amount of global liquidity, reduces the benefits of regulation for the other country. Finally the model delivers the result that, starting from the laissez-faire equilibrium, a unilateral adoption of liquidity regulation can be welfare reducing for the regulated country and welfare improving for the unregulated country. These results complement the paper's main results about policy coordination and provide additional ways to look at the public goods problem inherent to liquidity regulation in an international context.

Even though the bulk of the analysis is conducted in the context of a model where agents borrow internationally when hit by liquidity shocks, we show that all results carry over to a closely related setup where agents raise liquidity during crises by selling long-term assets rather than by borrowing. In that alternative setup, the pecuniary externality works through an asset price and takes the form of cash-in-the-market pricing and fire-sale externalities (see Allen and Gale 1998). The coordination problem between national regulators emphasized in this paper can hence be alternatively interpreted as arising from a failure to mutually commit to supporting asset prices during crises.

At an abstract level, the regulation game analyzed in this paper corresponds to a liquidity demand model with two large agents. It is often argued that market power mitigates the harm caused by systemic externalities in financial markets, because large agents partly internalize the effect of their actions on prices. We show here that this need not be the case. Whether market power attenuates or amplifies the distortions caused by market incompleteness crucially depends on the direction of the pecuniary externalities imposed on ex-post identical agents. Market power tends to attenuate market incompleteness distortions when these externalities are negative (like fire-sale externalities), but it tends to amplify distortions when the externalities are positive (like cash-in-the-market pricing externalities). In situations similar to our model, where agents impose both negative and positive externalities on ex-post identical agents, the effect of market power on market incompleteness distortions can potentially go either way.

Literature

The paper belongs to a recent research agenda that motivates financial regulation from a second best perspective in incomplete markets environments. It is most closely related to the liquidity regulation approach of Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009), and to the sudden stop prevention analysis of Caballero and Krishnamurthy (2001,

2004) for emerging countries. As in these papers, the market failure calling for government intervention in the present paper originates from a pecuniary externality operating on a spot market for liquidity.[2] But our paper stands out from these by explicitly formulating a multi-country framework. The model structure is therefore closer to that of Castiglionesi, Feriozzi, and Lorenzoni (2010), whose main focus is on the positive implications of financial integration for the liquidity of banks' portfolios and the magnitude of interest rate spikes in crises. They briefly touch upon normative issues by solving numerically for the constrained efficient allocation, but ignore the potential coordination problem that arises in attempting to implement this allocation when regulation is set at a national level. In contrast, we consider the incentives faced by rationally acting national regulators, analytically characterize the equilibrium of the non-cooperative policy game and compare it with both the laissez-faire outcome and the constrained efficient allocation achieved by coordinating liquidity regulation globally.

In modeling the strategic interaction among national governments, this paper follows the game theoretic approach to macroeconomic policy coordination pioneered by Hamada (1974, 1976, 1979).[3] Governments are assumed to set policies to maximize national welfare, while taking into account their power to affect international prices. By attempting to use market power to shift surplus in favor of domestic residents, governments generally end up in Pareto-inferior equilibria. Our results parallel those of studies where distortions induced by openness can overturn the direction of a desirable policy intervention, such as Corsetti and Pesenti (2001). In their macroeconomic model, monopoly distortions in production together with nominal rigidities make unexpected monetary expansions welfare improving (as in Blanchard and Kiyotaki 1987 and Ball and Romer 1989) when carried out simultaneously at home and abroad. But the same policy can be welfare reducing for a country acting in isolation because of adverse endogenous terms-of-trade movements. Similarly, in our model, financial market imperfections make prudential regulation unambiguously welfare improving when introduced

[2] Lorenzoni (2008) and Korinek (2011b) emphasize similar pecuniary externalities working through asset prices and fire-sale spirals during financial crises. Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013), Jeanne and Korinek (2011), Bianchi and Mendoza (2010) and Bengui (2011) investigate the quantitative relevance of these pecuniary externalities in infinite horizon macroeconomic models.

[3] Since Hamada's work, there has been a large literature on macroeconomic policy coordination and interdependence. See the reviews in Cooper (1985) and Persson and Tabellini (1995).

jointly at home and abroad. But terms-of-trade movements working through the interest rate during crises and associated spillover effects can make the unilateral introduction of such a policy welfare reducing for a given country.

Our paper is also related to Acharya (2003), who studies the consequences of an international convergence of bank capital requirements in the presence of heterogenous national closure policies, and Dell Arricia and Marquez (2006), who analyze the incentives for national bank regulators to form a regulatory union. In both of those papers, regulatory spillovers operate through changes in the degree of competition faced by banks on international loan markets during tranquil times. In our model, this competition channel is absent and international spillovers only arise through pecuniary externalities operating in a global market for liquidity during financial crises. In abstracting from the competition channel, our analysis shares some features with that of Morrison and White (2009), who analyze the costs and benefits of harmonizing capital requirements in a context where national regulators have heterogenous bank auditing capabilities.

Finally, our paper is related to a recent literature on optimal capital controls. Korinek (2013) argues that despite causing spillover effects, capital controls designed by atomistic countries to correct for domestic externalities need not be coordinated internationally. However, when countries are large, as in Costinot, Lorenzoni, and Werning (2012), optimal capital controls respond to dynamic terms-of-trade manipulation motives and lead to suboptimal outcomes, as in the optimal tariffs literature. In our analysis, the case for international coordination arises cumulatively from a need to correct for global (as opposed to domestic) externalities *and* a state-contingent dynamic terms-of-trade manipulation motive due to the presence of large countries.

The paper is structured as follows. The model is presented in section 2. Global and national liquidity regulations are analyzed in section 3. Section 4 works out the implications of the framework for the international spillovers of national liquidity standards. Section 5 presents an alternative model in which the ex-post intermediation of funds occurs via an asset market rather than via a credit market. Section 6 concludes. All proofs are collected in the Appendix.

2 A two-country model of liquidity demand

This section presents the model in which the coordination problem between national regulators is analyzed. The environment, inspired by Caballero and Krishnamurthy (2001), is specified in section 2.1, and the competitive equilibrium is characterized in section 2.2.

2.1 Preferences, technology and markets

Agents, Time and Preferences The world economy is composed of two countries, indexed by $j \in \{A, B\}$, that are ex-ante identical with respect to preferences, endowments and technology. Each country is populated by a continuum of identical agents. Time lasts for three periods $t = 0, 1, 2$, and consumption takes place at date 2. Agents' preferences over date 2 consumption are represented by an increasing, concave and twice continuously differentiable utility function $u(\cdot)$.

Technology Each agent is born with an endowment of one unit of the consumption good at date 0, and decides how to allocate this endowment between investment in a risky and illiquid project k and investment in a safe storage technology ℓ. With probability $1 - \alpha$, all projects in country j remain *intact*. An intact project does not require additional funds at date 1 and yields a date 2 return of $A > 1$. With probability α, a project becomes *distressed* and necessitates a renewed investment of one good per unit at date 1 for the project to yield a date 2 return of A. Each unit not shored up at date 1 yields a reduced date 2 return of $r < 1$. Distressed agents have the possibility to scale down investment at date 1. For an initial project of size k, a continuation scale θ results in a date 1 cost of θk and a date 2 return of $rk + \theta \Delta k$, where $\Delta \equiv A - r$. The storage technology yields one date $t + 1$ good per date t unit invested, and can be accessed both at date 0 and date 1.

Uncertainty All the uncertainty is resolved at date 1. Liquidity shocks are imperfectly correlated across the two countries. The sample space is given by $\Omega = \{(i, i), (i, s), (s, i), (s, s)\}$, where in state (i, s) country A is intact (i) and country B is distressed (s). The probability mass function $\pi : \Omega \to [0, 1]$ assigns a probability to each state $\omega \in \Omega$.

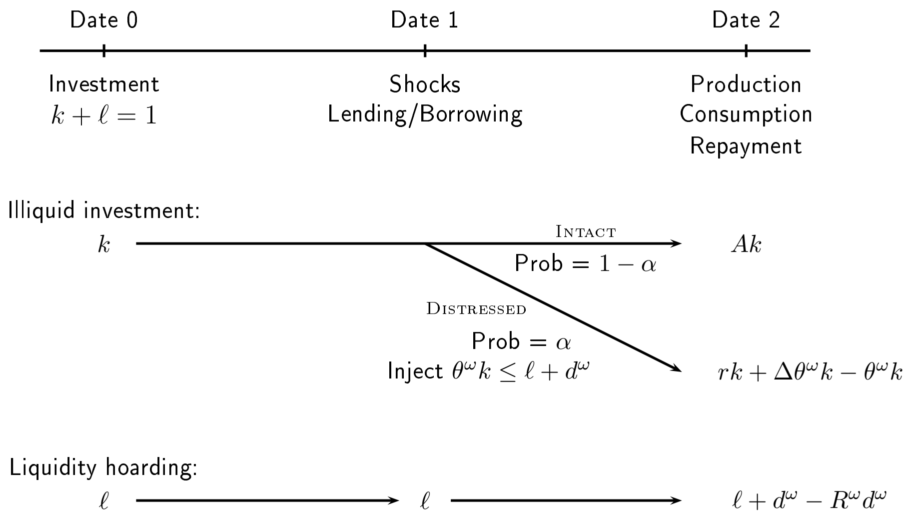

Figure 1: Time line.

Markets The imperfect correlation of liquidity shocks across countries creates opportunities for international risk-sharing. The assumption maintained throughout the paper, though, is that only non-state contingent bonds can be traded across borders. This can be motivated by a lack of verifiability of country-specific shocks.

Agents can partially substitute for risk-sharing by borrowing and lending at date 1 on an international spot market at interest rate R^ω.[4] d_j^ω denotes the amount borrowed by a country j agent. $-d_j^\omega$ accordingly denotes the amount lent. The size of loans on this market is constrained by distressed agents' ability to commit to repay their debt at date 2. We assume that lenders can only repossess a fraction κ of distressed agents' date 2 output and that any excess production based on reinvestment at date 1 is neither observable nor verifiable. This implies that borrowing must satisfy the collateral constraint $R^\omega d_j^\omega \leq \kappa r k_j$. The time line is represented in Figure 1.

Assumptions on parameters

[4]Because agents are ex-ante identical, non-contingent claims would not be traded at date 0 in equilibrium, so the date 0 market for these claims can be abstracted from without loss of generality.

Assumption 1 (Yield of illiquid project)**.** *The yield on the illiquid project satisfies* $1 < A \leq \frac{3}{2}$ *and* $\Delta \equiv A - r > 1$.

The assumption that $1 < A \leq \frac{3}{2}$ ensures that the illiquid project has a higher yield than the liquid asset in normal times, albeit not excessively so. The assumption that $\Delta > 1$, on the other hand, captures the idea that distressed firms have a high marginal value of investment during crisis times. It implies that shoring up an additional unit of a distressed project, if feasible, is always socially desirable.

Assumption 2 (Probability of crisis)**.** *The probability* α *of a project becoming distressed satisfies* $\underline{\alpha} < \alpha < \bar{\alpha}$, *where* $\underline{\alpha}$ *is given by*

$$\underline{\alpha} = \frac{(A-1)u'(A)}{(A-1)u'(A) + (\Delta - r)u'(r)}$$

and $\bar{\alpha}$ *is the smallest positive root of the quadratic equation*

$$(1-\alpha)^2(A-1)u'\Big(\frac{2A}{3} + \frac{1}{3}\Big) + \alpha(1-\alpha)(A-\Delta)u'\Big(\frac{2A}{3} + \frac{\Delta}{3}\Big) + \alpha(r-\Delta)u'\Big(\frac{2r}{3} + \frac{\Delta}{3}\Big) = 0.$$

Assumption 2 captures the fact that financial crises are low probability events, but that they are likely enough to induce precautionary behavior in the form of some liquidity hoarding in the competitive equilibrium of the model. $\alpha > \underline{\alpha}$ guarantees that crises are likely enough that agents find it optimal to hoard a positive amount of liquid assets, while $\alpha < \bar{\alpha}$ ensures that crisis are rare enough that when a crisis hits one of the two countries, the global aggregate amount of liquidity hoarded ex-ante is not sufficient to shore up all illiquid projects in the distressed country. Together, these two assumptions imply that there is partial liquidation in a crisis.

2.2 Competitive equilibrium

We start by considering equilibrium in the date 1 spot market for given date 0 decisions. We will then proceed backwards to solve for the competitive equilibrium and regulated equilibria at date 0.

2.2.1 Date 1 spot market equilibrium

The date 1 value of an intact agent in country j is given by

$$V_i^\omega(k_j, \ell_j) \equiv \max_{0 \leq -d_j^\omega \leq \ell_j} u\Big(Ak_j - d_j^\omega(R^\omega - 1) + \ell_j\Big). \tag{1}$$

The intact agent's date 2 consumption in (1) is the sum of the return on its illiquid project Ak_j, the return on the loan made on the date 1 spot market $-R^\omega d_j^\omega$, and the return on the funds invested at date 1 in the storage technology $\ell_j + d_j^\omega$. We assume that intact agents can only lend on the date 1 spot market.[5] Their lending capacity is limited by their date 1 liquid resources ℓ_j.

The form of the objective in (1) implies a simple date 1 loan supply schedule for intact agents. For $R^\omega < 1$, intact agents do not want to lend at all. At $R^\omega = 1$, they are indifferent between lending any amount between 0 and ℓ_j. Finally, when $R^\omega > 1$ they are willing to lend all their available liquid resources ℓ_j. The loan supply schedule of intact agents is displayed in the left panel of Figure 2.

The date 1 value of a distressed agent in country j is given by

$$V_s^\omega(k_j, \ell_j) \equiv \max_{\theta_j^\omega, d_j^\omega} u\Big(rk_j + \Delta\theta_j^\omega k_j - \theta_j^\omega k_j + \ell_j - (R^\omega - 1)d_j^\omega\Big) \tag{2}$$

subject to

$$\theta_j^\omega k_j \leq \ell_j + d_j^\omega \tag{3}$$

$$R^\omega d_j^\omega \leq \kappa r k_j \tag{4}$$

$$\theta_j^\omega \leq 1 \tag{5}$$

A distressed agent's date 2 consumption in (2) is the sum of the return on its illiquid project $rk_j + \Delta\theta_j^\omega k_j$ and the return on the funds invested at date 1 in the storage technology $\ell_j + d_j^\omega - \theta_j^\omega k_j$, minus the debt repayment $R^\omega d_j^\omega$. (3) is the date 1 budget constraint indicating that the reinvestment $\theta_j^\omega k_j$ needs to be covered by the liquid resources $\ell_j + d_j^\omega$. (4) is the

[5] This assumption is without loss of generality, because intact agents would only strictly prefer to borrow funds if $R^\omega < 1$, which cannot occur in equilibrium.

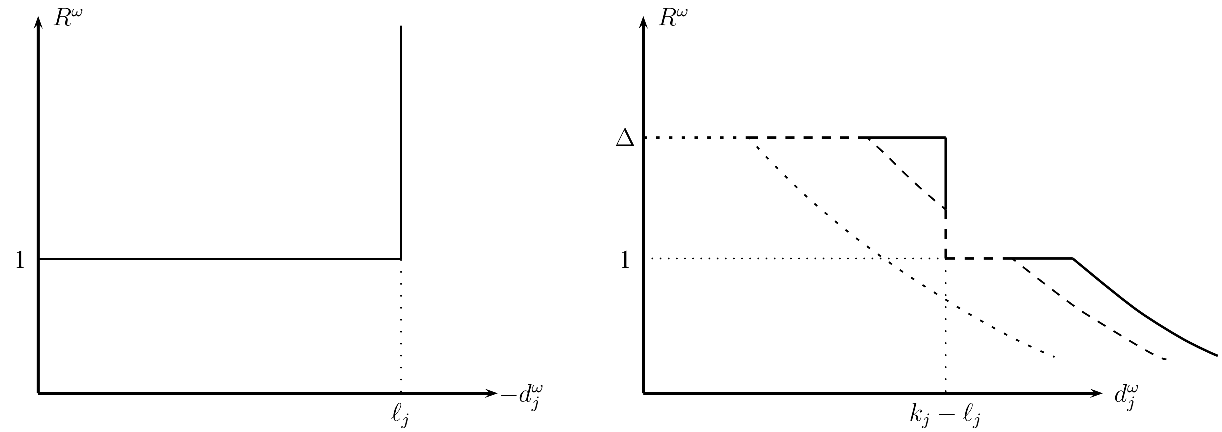

Figure 2: Supply by intact (left) and demand by distressed (right) on date 1 spot market.

collateral constraint, and (5) indicates that investment cannot be scaled up at date 1.

Given the assumption that $\Delta > 1$ and the fact that θ_j^ω and d_j^ω enter additively in the expression for consumption, the loan demand and optimal continuation scale of distressed agents take simple forms. For $R^\omega < 1$, the agents hit their collateral constraint (4). For $1 \leq R^\omega < \Delta$, they borrow the minimum of the amount they need to salvage all their assets, $k_j - \ell_j$, and their borrowing limit, $\kappa r k_j / R^\omega$. At $R^\omega = \Delta$, they are indifferent between borrowing any amount between 0 and $\min\left\{k_j - \ell_j, \kappa r k_j/\Delta\right\}$. Finally, for $R^\omega > \Delta$, they do not want to borrow at all. The optimal continuation scale θ_j^ω is accordingly given by $\min\left\{1, \frac{\ell_j}{k_j} + \frac{\kappa r}{R^\omega}\right\}$ for $R^\omega < \Delta$, by any amount between $\frac{\ell_j}{k_j}$ and $\min\left\{1, \frac{\ell_j}{k_j} + \frac{\kappa r}{\Delta}\right\}$ when $R^\omega = \Delta$, and by $\min\left\{1, \frac{\ell_j}{k_j}\right\}$ for $R^\omega > \Delta$. The loan demand schedule of distressed agents is displayed in the right panel of Figure 2. The loan demand curve is drawn for various values of κ, with the curves to the left associated with a lower κ. The collateral constraint is binding on the downward-sloping parts of the demand curve, and is non-binding elsewhere.

In states of the world where the two countries are both distressed or both intact, the date 1 spot market equilibrium is trivial. In state (i, i), the equilibrium is simply given by $R^\omega = 1$ and $d_A^\omega = d_B^\omega = 0$. In state (s, s), it is given by $R^\omega = \Delta$ and $d_A^\omega = d_B^\omega = 0$, provided that $\ell_A, \ell_B < \frac{1}{2}$[6]. However, in states of the world where one country is intact and the other is distressed, i.e. in (i, s) and (s, i), the spot market equilibrium can a priori fall in four distinct regions, depending on where the loan demand and loan supply curves intersect (similarly to

[6]Assumptions 1 and 2 are sufficient for this latter condition to be satisfied in the date 0 equilibrium.

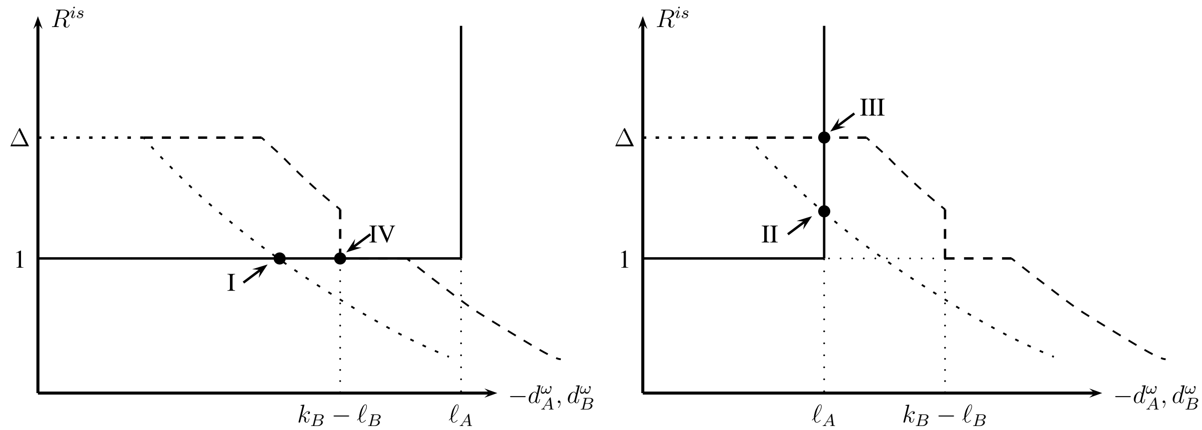

Figure 3: Regions for spot market equilibrium in state (i, s).

		Global liquidity	
		ample	*scarce*
Distressed country	*constrained*	region I	region II
	unconstrained	region IV	region III

Table 1: Regions for equilibrium in state (i, s) or (s, i).

Caballero and Krishnamurthy 2001).

These four types of equilibria are displayed in Figure 3.[7] As represented in Table 1, the regions can be categorized according to (a) whether global liquidity is ample or scarce, and (b) whether the distressed country's collateral constraint is non-binding or binding. When global liquidity is ample, i.e. when $\ell_A + \ell_B \geq k_B$ (regions I and IV), the equilibrium interest rate is low: $R^{is} = 1$. In region I, the distressed country is constrained, and there is partial liquidation ($\theta_B^{is} = (\kappa r k_B + \ell_B)/k_B < 1$). In region IV, the distressed country is unconstrained, and borrowing is high enough to allow for full continuation and avoid partial liquidation ($\theta_B^{is} = 1$). When, on the other hand, global liquidity is scarce, i.e. when $\ell_A + \ell_B < k_B$ (regions II and III), the equilibrium interest rate is high: $R^{is} > 1$. In these

[7] The equilibria are drawn for state (i, s), where country A is intact and country B is distressed, but equilibria in state (s, i) take identical forms, with the subscripts A and B interchanged.

regions, all the global liquidity is used to shore up the distressed country's assets, but the aggregate shortage of date 1 resources results in partial liquidation ($\theta_B^{is} = (\ell_A + \ell_B)/k_B < 1$). In region II, the distressed country is constrained and can only pledge to offer to lenders a return $R^{is} = \kappa r k_B/\ell_A$, lower than the social marginal return Δ. In region III, on the other hand, the distressed country is unconstrained and the lender country can be compensated at the social marginal return of liquidity in consumption goods terms, $R^{is} = \Delta$. As is common in models with borrowing constraints (i.e. Bernanke and Gertler 1989, Kiyotaki and Moore 1997), a wedge between the internal and external rate of return on investment arises in regions I and II.

The relevant region for equilibrium on the date 1 spot market in states (i, s) and (s, i) depends on date 0 choices in the two countries. The next section provides conditions on parameters under which date 0 choices in a competitive equilibrium lead to particular regions. For reasons made explicit below, the focus of the analysis will henceforth be on parameter configurations which result in the competitive equilibrium falling in region II.

2.2.2 Date 0 equilibrium

At date 0, an agent in country j takes the schedule of date 1 interest rates R^ω as given and solves

$$\max_{k_j, \ell_j} \sum_{\omega \in \Omega} \pi^\omega V^\omega(k_j, \ell_j) \tag{6}$$

subject to

$$k_j + \ell_j = 1, \tag{7}$$

where

$$V^\omega(k_j, \ell_j) \equiv \begin{cases} V_i^\omega(k_j, \ell_j) & \text{if country } j \text{ is intact} \\ V_s^\omega(k_j, \ell_j) & \text{if country } j \text{ is distressed} \end{cases} \tag{8}$$

with V_i^ω and V_s^ω defined in (1) and (2), respectively. The first-order condition

$$\sum_{\omega \in \Omega} \pi^\omega \frac{\partial V^\omega(k_j, \ell_j)}{\partial k_j} = \sum_{\omega \in \Omega} \pi^\omega \frac{\partial V^\omega(k_j, \ell_j)}{\partial \ell_j} \tag{9}$$

together with the date 0 budget constraint (7) characterize the agent's optimal choice. A *competitive equilibrium* (CE) of the model consists of date 0 decisions $(k_j, \ell_j)_{j\in\{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices, the decisions solve the problems in (6-7), (1) and (2); and (b) markets clear.[8] In what follows, the values of k and ℓ in a symmetric competitive equilibrium are denoted by k^{CE} and ℓ^{CE}.

Assumption 2 guarantees that the probability of a crisis is not large enough to produce a situation where the aggregate amount of liquidity set aside in a symmetric competitive equilibrium, $2\ell^{CE}$, is sufficient to avoid any liquidation in the states of the world where one country is intact and the other is distressed. In other words, assumption 2 ensures that $k^{CE} > 2/3$. This implies that we can focus on situations in which only regions I, II or III in states (i, s) and (s, i) can arise in equilibrium.

To gain further insights into the properties of a competitive equilibrium of the model, it is useful to look at the agents' date 1 value function. In state ω, the value function is given by

$$V_i^\omega(k_j, \ell_j) = u\Big(Ak_j + R^\omega \ell_j\Big) \tag{10}$$

for an intact agent, and by

$$V_s^\omega(k_j, \ell_j) = u\Big(rk_j + (\Delta - R^\omega)\frac{\kappa r k_j}{R^\omega} + \Delta \ell_j\Big) \tag{11}$$

for a distressed agent. The terms in (10) are straightforward to interpret. For an intact agent, the illiquid asset yields a return of A, while the liquid asset yields a return of R^ω, with $1 \leq R^\omega \leq \Delta < A$. The marginal value of the illiquid asset in terms of date 2 consumption is thus higher than that of the liquid asset. The terms in (11) are similarly straightforward. For a distressed agent, a unit of the illiquid asset yields a baseline return of r, plus a net return of $\Delta - R^\omega$ on the $\kappa r / R^\omega$ of external financing raised against collateral, whereas a unit of the liquid asset allows the continuation of one unit of the investment project, thereby yielding a return of Δ. The marginal value of the liquid asset is thus higher than that of the illiquid asset.[9] Given concave utility, from the perspective of period 0 the illiquid asset is a

[8] The continuation scale of an intact agent is by definition set to $\theta_j^\omega = 1$.

[9] The marginal value of the illiquid asset for a distressed agent is the highest when $R^\omega = 1$, in which case it is given by $r + (\Delta - 1)\kappa r = [(1-\kappa) + \kappa\Delta]r \leq \Delta r < \Delta$.

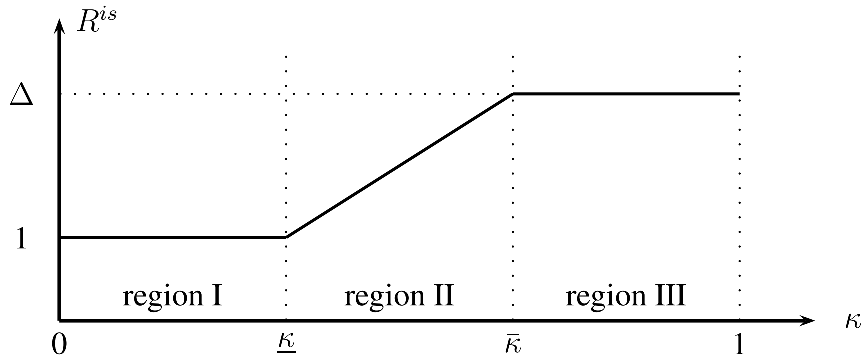

Figure 4: Equilibrium interest rate in state (i, s) as a function of κ, in competitive equilibrium.

bad hedge, while the liquid asset is a good hedge.

The equilibrium can be further characterized as falling into one of the aforementioned three regions, depending on the tightness of financial constraints κ. In particular, there are thresholds $\underline{\kappa}$ and $\bar{\kappa}$, such that

- for $\kappa < \underline{\kappa}$, the symmetric competitive equilibrium leads to region I, i.e. $\frac{2}{3} < k^{CE} < \frac{1}{1+\kappa r}$,
- for $\underline{\kappa} \leq \kappa \leq \bar{\kappa}$, the symmetric competitive equilibrium leads to region II, i.e. $\frac{1}{1+\kappa r} \leq k^{CE} \leq \frac{\Delta}{\Delta+\kappa r}$,
- for $\kappa > \bar{\kappa}$, the symmetric competitive equilibrium leads to region III, i.e. $\frac{\Delta}{\Delta+\kappa r} < k^{CE} < 1$.

Hence, very tight financial constraints lead to region I, mildly tight constraints lead to region II, and loose constraints lead to region III. The interest rate in states (i, s) and (s, i) is pictured as a function of the tightness of financial constraints in Figure 4.

For the remainder of the paper, we focus on the case in which the symmetric competitive equilibrium leads to region II, via the following assumption.

Assumption 3 (Tightness of financial constraints)**.** *The tightness of financial constraints* κ *satisfies* $\underline{\kappa} < \kappa < \bar{\kappa}$.

Assumption 3 ensures that the symmetric competitive equilibrium leads to the interior of region II, i.e. that $\frac{1}{1+\kappa r} < k^{CE} < \frac{\Delta}{\Delta+\kappa r}$. A focus on region II is warranted by the fact that in that region, the date 1 spot market equilibrium provides a stylized description of actual liquidity crises in two key respects: (a) the aggregate shortage of liquidity results in a spike in the cost of borrowing, and (b) the pervasiveness of financial constraints causes a wedge between the internal and external marginal value of funds for distressed entities. Importantly, in this region, the price of liquidity (i.e. the interest rate) is a locally decreasing function of the amount of liquidity set aside ex-ante by lenders, and a locally increasing function of the amount of illiquid collateral owned by borrowers. Equilibrium in this region therefore captures the key intuition that the price of liquidity in crises decreases with the supply of it and increases with the demand for it. A lower ex-ante illiquid investment scale decreases the interest rate in crises, which benefits distressed borrowers more than it hurts intact lenders at the margin. Due to the incompleteness of markets, this pecuniary externality causes a market failure: the date 0 choices in a competitive equilibrium are not constrained efficient, as perturbating these allocations locally has first-order welfare effects via shifts in interest rates. The failure of the competitive equilibrium to be constrained efficient motivates the analysis of prudential regulation from a second best perspective.

3 Efficiency and planning problems

In assessing the welfare properties of competitive equilibria in incomplete markets economies, one is generally interested in whether the market system allocates resources efficiently given the set of markets operating (see Stiglitz 1982). This section analyzes alternative allocation mechanisms and compares their welfare properties with those of the decentralized equilibrium described in section 2.2. Sections 3.1 and 3.2 characterize the allocations resulting from global and national planners making date 0 investment decisions subject to the same set of enforcement and informational frictions as private agents. These allocation mechanisms are interpreted as regulated equilibria, as in Allen and Gale (2004).

3.1 Constrained global planner

Given the presumed failure of the first welfare theorem, it is natural to ask how a constrained social planner would want to regulate date 0 investment decisions. To this end, we start by considering a global planner who maximizes the sum of agents' expected utility in the two countries, makes date 0 investment decisions on the behalf of private agents in both countries, and lets the spot market operate competitively at date 1. Importantly, the planner is assumed to be subject to the same set of informational and enforcement constraints as the private sector, in that he cannot implement transfers contingent on the realization of date 1 uncertainty.

Since agents are ex-ante identical, the planner is assumed to assign equal weights to agents in the two countries when making date 0 choices. The planner's date 1 value function $\tilde{V}^{\omega}(k_A, k_B, \ell_A, \ell_B)$ is given by the sum of the respective expressions in (10) and (11) in which the equilibrium interest rate has been substituted in. When the interest rate is not an explicit function of date 0 choices, as in states (i,i), (s,s), and $(i,s)/(s,i)$ in region I, III and IV, $\tilde{V}^{\omega}(k_A, k_B, \ell_A, \ell_B)$ coincides with the sum of the value functions perceived by the agents in the two countries. But when the interest rate depends explicitly on date 0 choices, as in states $(i,s)/(s,i)$ in region II, the planner's value function is given by

$$\tilde{V}^{is}(k_A, k_B, \ell_A, \ell_B) = u\Big(Ak_A + R(k_B, \ell_A)\ell_A\Big) + u\Big(rk_B + \frac{[\Delta - R(k_B, \ell_A)]\kappa r}{R(k_B, \ell_A)}k_B + \Delta\ell_B\Big) \quad (12)$$

$$\tilde{V}^{si}(k_A, k_B, \ell_A, \ell_B) = u\Big(rk_A + \frac{[\Delta - R(k_A, \ell_B)]\kappa r}{R(k_A, \ell_B)}k_A + \Delta\ell_A\Big) + u\Big(Ak_B + R(k_A, \ell_B)\ell_B\Big) \quad (13)$$

with $R(k_j, \ell_{-j}) \equiv \frac{\kappa r k_j}{\ell_{-j}}$. Comparing the expressions in (12-13) with those in (10-11), it is apparent that the global planner's marginal valuation of the two assets does generally not coincide with the private marginal valuations in states (i,s) and (s,i). This is stated more precisely in the following lemma.

Lemma 1 (Assets' valuation of global planner vs. private agents)**.** *In region II, the global planner values*

1. *the intact country's liquid asset more highly than private agents in the state of nature where the other country is distressed, and*

2. *the distressed country's illiquid asset less highly than private agents in the state of nature where the other country is intact.*

The two results in lemma 1 arise from the fact that the global planner internalizes the effect of the two countries' asset positions on the interest rate in states where there is trade on the date 1 loan market, while private agents take this price as given. The results follow from partially differentiating (10-11) and (12-13). Without loss of generality, consider state (i, s). The global planner's valuation of the intact country's liquid asset in the state of nature where the other country is distressed is

$$\frac{\partial \tilde{V}^{is}}{\partial \ell_A} = R^{is} {u'_A}^{is} + \ell_A \frac{\partial R^{is}}{\partial \ell_A} \left[{u'_A}^{is} - \frac{\Delta}{R^{is}} {u'_B}^{is} \right], \tag{14}$$

where ${u'_A}^{is} \equiv u'\left(Ak_A + R^{is}\ell_A\right)$ and ${u'_B}^{is} \equiv u'\left(rk_B + (\Delta - R^{is})\frac{\kappa r k_B}{R^{is}} + \Delta \ell_B\right)$ are the marginal utilities of intact and distressed agents, respectively. As shown in the Appendix, distressed agents have a higher marginal utility than intact agents in region II, i.e. ${u'_B}^{is} > {u'_A}^{is}$. The expression in (14) reveals that the global social marginal value of a unit of the liquid asset in the hands of intact agents, in the state of nature where the other country is distressed, consists of two terms. The first term coincides with the private marginal valuation $\frac{\partial V_i^{is}}{\partial \ell_A} = R^{is}{u'_A}^{is}$. The second term represents the global planner's valuation of the redistribution of wealth brought about by the interest rate decrease associated to a marginally higher amount of liquid assets held by the intact country. This valuation is the product of three components: the base of the wealth redistribution ℓ_A, the price effect $\frac{\partial R^{is}}{\partial \ell_A}$ and the wedge in the marginal values of wealth $\left[{u'_A}^{is} - \frac{\Delta}{R^{is}}{u'_B}^{is}\right]$. The base corresponds to the amount lent by the intact country to the distressed country, and is positive. The price effect embodies the effect of a larger supply of liquidity on the interest rate, and is negative. Lastly, the wedge between the marginal values of wealth is negative, both because the marginal utility of consumption is higher for the distressed country than for the intact one (${u'_B}^{is} > {u'_A}^{is}$), and because the distressed country's internal rate of return is larger than the market interest rate ($\Delta > R^{is}$). Hence, the global planner's valuation of the intact country's liquid asset is higher than the private agent's valuation thereof, because the planner values positively the transfer of wealth from the intact country to the distressed country induced by a marginally higher supply of liquidity on the date 1 loan market.

For part 2, the private valuation is $\frac{\partial V_s^{is}}{\partial k_B} = \left[r + (\Delta - R^{is})\frac{\kappa r}{R^{is}}\right] {u'_B}^{is}$ while the global planner's valuation is

$$\frac{\partial \tilde{V}^{is}}{\partial k_B} = \left[r + (\Delta - R^{is})\frac{\kappa r}{R^{is}}\right] {u'_B}^{is} + \ell_A \frac{\partial R^{is}}{\partial k_B}\left[{u'_A}^{is} - \frac{\Delta}{R^{is}}{u'_B}^{is}\right]. \tag{15}$$

As was the case for the liquid asset, the expression in (15) reveals that the global social marginal value of an illiquid asset in the hands of distressed agents, in the state of nature where the other country is intact, consists of two terms. The first term coincides with the private marginal valuation, while the second term reflects the global planner's valuation of the redistribution of wealth brought about by the interest rate increase associated to a marginally higher amount of illiquid assets held by the distressed country. This valuation is again the product of three components: the base of the wealth redistribution ℓ_A, the price effect $\frac{\partial R^{is}}{\partial k_B}$, and the wedge in the marginal values of wealth $\left[{u'_A}^{is} - \frac{\Delta}{R^{is}}{u'_B}^{is}\right]$. This time the price effect is positive: more collateralizable wealth for distressed borrowers increases the demand for loans and pushes the interest rate up. Hence, the global planner's valuation of the distressed country's illiquid asset is lower than the private agent's valuation thereof, because the planner values negatively the transfer of wealth from the distressed country to the intact country induced by a marginally higher demand for liquidity on the date 1 loan market.

The differential asset valuation result emphasized in lemma 1 leads the planner to make date 0 investment choices that generally differ from the ones that arise in a competitive equilibrium. The global planner solves

$$\max_{(k_j,\ell_j)_{j\in\{A,B\}}} \sum_{\omega\in\Omega} \pi^\omega \tilde{V}^\omega(k_A, k_B, \ell_A, \ell_B) \tag{16}$$

subject to

$$k_j + \ell_j = 1 \qquad \text{for} \quad j \in \{A, B\}. \tag{17}$$

In other words, the planner makes date 0 choices while anticipating the effect of its decisions on the determination of the spot market equilibrium at date 1. A *globally regulated equilib-*

rium (GRE) consists of date 0 decisions $(k_j, \ell_j)_{j\in\{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices, the private sector's date 1 decisions solve the problems in (1) and (2); (b) the global planner's date 0 decisions solve the problem in (16); and (c) markets clear. For future reference, the levels of k and ℓ chosen by a global planner in a (symmetric) GRE are denoted by $\tilde{k}$ and $\tilde{\ell}$.

How does $\tilde{k}$ relate to k^{CE}? For $0 \leq k < \frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r} < k \leq 1$, the planner's objective coincides with the private agents' objectives, since in these regions (I, III and IV), the interest rate is not locally sensitive to date 0 choices. Furthermore, under assumptions 1, 2 and 3, the investment choice k^{CE} in a competitive equilibrium falls in the interior of the interval $[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}]$, while the private agents' objective is monotonically increasing over $\left[0, \frac{1}{1+\kappa r}\right)$ and monotonically decreasing over $\left(\frac{\Delta}{\Delta+\kappa r}, 1\right]$. Since over these latter two intervals, the planner's and private agents' objectives are the same, the planner's objective must be monotonically increasing over $\left[0, \frac{1}{1+\kappa r}\right)$ and monotonically decreasing over $\left(\frac{\Delta}{\Delta+\kappa r}, 1\right]$. The global planner's optimal investment choice $\tilde{k}$ therefore has to fall in the interval $[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}]$ (region II). $\tilde{k}$ thus necessarily satisfies

$$\sum_{\omega\in\Omega} \pi^\omega \frac{\partial \tilde{V}^\omega(\tilde{k}, \tilde{k}, 1-\tilde{k}, 1-\tilde{k})}{\partial k_j} - \sum_{\omega\in\Omega} \pi^\omega \frac{\partial \tilde{V}^\omega(\tilde{k}, \tilde{k}, 1-\tilde{k}, 1-\tilde{k})}{\partial \ell_j} \lesseqgtr 0 \qquad \text{for} \quad j \in \{A, B\}, \tag{18}$$

with "$\leq$" if $\tilde{k} = \frac{1}{1+\kappa r}$, with "$=$" if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{1}{1+\kappa r}$ and with "$\geq$" if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$. We are now in a position to compare $\tilde{k}$ with k^{CE}.

Proposition 1 (GRE vs. CE)**.** *A globally regulated equilibrium features a more liquid investment portfolio than the competitive equilibrium, i.e.* $\tilde{k} < k^{CE}$ *and* $\tilde{\ell} > \ell^{CE}$*.*

Proposition 1 provides a characterization of the constrained efficient date 0 investment choice, and establishes the constrained inefficiency of the decentralized equilibrium. At date 1, the global planner's and private agents' valuations of the liquid asset coincide in all states of nature, except in the one where agents lend to distressed foreigners, in which case the planner's valuation is strictly higher. Similarly, the global planner's and private agents' valuations of the illiquid asset coincide in all states, except in the one where agents borrow from intact foreigners, in which case the planner's valuation is strictly lower. These wedges between the private and social valuations of the respective assets naturally lead the global planner to invest more in the liquid asset and less in the illiquid asset at date 0.

Since agents in both countries are ex-ante identical, the welfare metric is unambiguously given by a representative agent's expected utility at date 0. This criterion corresponds to the global planner's objective, re-scaled by 1/2. Since the planner's objective is strictly decreasing in k for $k \geq \tilde{k}$, welfare is strictly higher in a GRE than in the CE.[10] As emphasized in lemma 1, the global planner recognizes that, in the relevant region, a marginally more liquid investment portfolio at date 0 brings about a redistribution of wealth from intact lenders to distressed borrowers at date 1 in states (i, s) and (s, i). From an ex-ante perspective, such a redistribution of wealth is welfare improving. This leads the global regulator to tilt investment portfolios in favor of the liquid asset, as stated in proposition 1. The first welfare theorem fails because of pecuniary externalities and market incompleteness.

From a positive perspective, global regulation makes financial crises less severe in two respects. First, there is always less liquidation of long assets during a crisis with global regulation than in the laissez-faire case. In states of nature where one country is distressed and the other is intact, liquidation is given by $1 - \theta^{is} = 1 - \theta^{si} = 1 - 2(1 - k)/k$, while in the state where both countries are distressed, liquidation is given by $1 - \theta^{ss} = 1 - (1 - k)/k$. In both cases, $\tilde{k} < k^{CE}$ implies that there is less illiquid investment to shore up in a crisis and more available liquid resources to do so under global regulation than in a decentralized equilibrium. Second, global regulation results in less pronounced interest rate spikes when one country is hit and the other is not, since $R^{is} = R^{si} = \kappa r k/(1 - k)$. This is because with global regulation, the demand for funds is smaller and the supply of funds is larger, resulting in a milder increase in the price of liquidity in these states.

3.2 Constrained national planners

In order to understand the source of tensions that can arise in an environment where regulations are set independently in each country, we now consider the case of national planners who make date 0 decisions in their respective countries and let the spot market operate com-

[10] The fact that the planner's objective is strictly decreasing in k for $\tilde{k} \leq k \leq \frac{\Delta}{\Delta + \kappa r}$ follows from the planner's second-order condition.

petitively at date 1. The assumption that planners are subject to the same informational and enforcement frictions as private agents is maintained.

The national planners are assumed to maximize the expected utility of domestic agents when making date 0 choices. The country j planner's date 1 value function $\hat{V}^\omega(k_j, k_{-j}, \ell_j, \ell_{-j})$ is given by the expression in (10) or (11) in which the equilibrium interest rate has been substituted in. As was the case with the global planner, when the interest rate is locally sensitive to date 0 investment choices, the national planners' value function coincides with the private agents' value function.[11] But in states $(i, s)/(s, i)$ in region II, where the interest rate depends on date 0 choices locally, the planners' value functions are given by

$$\hat{V}_i^\omega(k_j, k_{-j}, \ell_j, \ell_{-j}) = u\Big(Ak_j + R(k_{-j}, \ell_j)\ell_j\Big), \tag{19}$$

$$\hat{V}_s^\omega(k_j, k_{-j}, \ell_j, \ell_{-j}) = u\Big(rk_j + \frac{[\Delta - R(k_j, \ell_{-j})]\kappa r}{R(k_j, \ell_{-j})}k_j + \Delta\ell_j\Big), \tag{20}$$

with $R(k_j, \ell_{-j}) \equiv \frac{\kappa r k_j}{\ell_{-j}}$. Comparing the expressions in (19-20) with those in (12-13) and (10-11), it is evident that the national planner's marginal valuation of the two assets does generally neither coincide with the global planner's marginal valuations, nor with the private marginal valuations in states (i, s) and (s, i). These wedges in valuations are established by the following two lemmas.

Lemma 2 (Assets' valuation of national planners vs. global planner)**.** *In region II, a national planner values*

1. *its intact agents' liquid asset less highly than the global planner in states of nature where the other country is distressed, and*
2. *its distressed agents' illiquid asset less highly than the global planner in states of nature where the other country is intact.*

Lemma 3 (Assets' valuation of national planners vs. private agents)**.** *In region II, a national planner values*

1. *its intact agents' liquid asset less highly than the agents themselves in states of nature where the other country is distressed, and*

[11] This is again the case in states (i, i), (s, s), and $(i, s)/(s, i)$ in regions I, III and IV

2. *its distressed agents' illiquid asset less highly than the agents themselves in states of nature where the other country is intact.*

The results in lemmas 2 and 3 are straightforward to interpret. Both the global planner and national planners internalize the effect of the two countries' asset positions on the interest rate in states where there is trade on the date 1 loan market, while private agents take this price as given. However, while the global planner weights welfare in the two countries equally, national planners only care about their domestic agents, effectively putting a zero weight on foreign welfare.

Formally, the results follow from partially differentiating (10-11), (12-13) and (19-20). Without loss of generality, consider state (i, s). The global planner's valuation of the intact country's liquid asset is given by (14). The intact country's national planner's valuation of its liquid assets, on the other hand, is

$$\frac{\partial \hat{V}_i^{is}}{\partial \ell_A} = R^{is} {u'_A}^{is} + \ell_A \frac{\partial R^{is}}{\partial \ell_A} {u'_A}^{is}. \tag{21}$$

As was the case with (14) for the global planner, the expression in (21) reveals that the intact country's national planner's marginal value of a unit of the liquid asset, in the state of nature where the other country is distressed, consists of two terms. The first term coincides with the private marginal valuation $\frac{\partial V_i^{is}}{\partial \ell_A} = R^{is} {u'_A}^{is}$. The second term represents the national planner's valuation of the redistribution of wealth brought about by the interest rate decrease associated to a marginally higher amount of liquid assets held by domestic residents. This valuation is the product of three components: the base of the wealth redistribution ℓ_A, the price effect $\frac{\partial R^{is}}{\partial \ell_A}$, and the domestic agent's marginal value of wealth ${u'_A}^{is}$. Since the first and third components are positive, and the second is negative, the second term in (21) is negative. This is in contrast with the global planner's valuation in (14). Since the intact country's national planner only cares about the welfare of domestic agents, it values negatively the transfer of wealth from domestic to foreign agents induced by a marginally higher supply of liquidity by its domestic residents on the date 1 loan market.

As for the second part of lemmas 2 and 3, the global planner's valuation of the intact country's illiquid asset is given by (14). The distressed country's national planner's valuation of its illiquid asset, on the other hand, is

$$\frac{\partial \hat{V}_s^{is}}{\partial k_B} = \left[r + (\Delta - R^{is}) \frac{\kappa r}{R^{is}} \right] {u'_B}^{is} + \ell_A \frac{\partial R^{is}}{\partial k_B} \left(-\frac{\Delta}{R^{is}} {u'_B}^{is} \right). \tag{22}$$

As was the case with (15) for the global planner, the expression in (22) reveals that the distressed country's national planner's marginal valuation of the illiquid asset, in the state of nature where the other country is intact, consists of two terms. The first term coincides with the private marginal valuation $\frac{\partial \hat{V}_s^{is}}{\partial k_B} = \left[r + (\Delta - R^{is}) \frac{\kappa r}{R^{is}} \right] {u'_B}^{is}$. The second term represents the national planner's valuation of the redistribution of wealth brought about by the interest rate increase associated to a marginally higher amount of illiquid assets held by domestic residents. This valuation is the product of three component: the base of the wealth redistribution ℓ_A, the price effect $\frac{\partial R^{is}}{\partial k_B}$, and the negative of domestic agents' marginal value of wealth $\frac{\Delta}{R^{is}} {u'_B}^{is}$. Since the first two components are positive and the third one is negative, the second term in (22) is negative. Note that this time, both the global planner and national planners value the illiquid asset less highly than private agents. However, their valuations do no coincide. Since the distressed country's national planner only cares about the welfare of domestic agents, it values the transfer of wealth from domestic to foreign agents induced by a marginally higher demand for liquidity by its domestic residents on the date 1 loan market more negatively than the global planner.

Given the differences in asset valuations between the national planners, the global planner and private agents, it is clear that regulation chosen at the national level will generally neither coincide with the laissez-faire, not with the constrained efficient allocation chosen by the global planner. Country j's national planner solves

$$\max_{k_j, \ell_j} \sum_{\omega \in \Omega} \pi^\omega \hat{V}^\omega (k_j, k_{-j}, \ell_j, \ell_{-j}) \tag{23}$$

subject to (7). National planners make date 0 choices while anticipating the effect of their decisions on the determination of the spot market equilibrium at date 1, and taking the action of the other country's national planner as given. A *nationally regulated equilibrium* (NRE) consists of date 0 decisions $(k_j, \ell_j)_{j \in \{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega \in \Omega, j \in \{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices and date 0 decisions, the private sector's date 1 decisions solve the problems in (1) and (2); (b) given (k_{-j}, ℓ_{-j}), (k_j, ℓ_j) solves the problem in (23); and (c) markets clear. The levels of k and ℓ chosen by national planners in a symmetric nationally regulated equilibrium are denoted by $\hat{k}$ and $\hat{\ell}$.

How does $\hat{k}$ relate to k^{CE} and $\tilde{k}$? We observe that under symmetric choices, for $0 \leq k < \frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r} \leq k \leq 1$, the national planners' objectives coincide with both the private agents' and the global planner's objective since in these regions the interest rate does not depend on date 0 choices locally. An argument analogous to that used in section 3.1 implies that the national planners' investment choices in a symmetric NRE have to fall in the interval $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$ (i.e. in region II). A necessary condition for a symmetric nationally regulated equilibrium is therefore given by

$$\sum_{\omega\in\Omega} \pi^\omega \frac{\partial \hat{V}^\omega(\hat{k}, \hat{k}, 1-\hat{k}, 1-\hat{k})}{\partial k_j} - \sum_{\omega\in\Omega} \pi^\omega \frac{\partial \hat{V}^\omega(\hat{k}, \hat{k}, 1-\hat{k}, 1-\hat{k})}{\partial \ell_j} \lesseqgtr 0 \qquad \text{for} \quad j \in \{A, B\}, \tag{24}$$

with "≤" if $\hat{k} = \frac{1}{1+\kappa r}$, with "=" if $\frac{1}{1+\kappa r} < \hat{k} < \frac{1}{1+\kappa r}$ and with "≥" if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. The following proposition establishes that date 0 investment choices by national planners are generally less liquid that those by the global planner.

Proposition 2 (NRE vs. GRE)**.** *A nationally regulated equilibrium features a weakly less liquid investment portfolio than a globally regulated equilibrium, i.e.* $\hat{k} \geq \tilde{k}$ *and* $\hat{\ell} \leq \tilde{\ell}$*. Furthermore, if* $\hat{k} > \frac{1}{1+\kappa r}$*, then a nationally regulated equilibrium features a strictly less liquid investment portfolio than a globally regulated equilibrium, i.e.* $\hat{k} > \tilde{k}$ *and* $\hat{\ell} < \tilde{\ell}$*.*

Lemma 2 had established that national planners, relative to the global planner, (a) undervalue liquid assets when their country is intact and the foreign country is distressed, and (b) undervalue illiquid assets when their country is distressed and the foreign country is intact. Proposition 2 states that unless both equilibria result in a left corner solution (within region II) for the date 0 investment choice (i.e. when $\hat{k} = \tilde{k} = \frac{1}{1+\kappa r}$), the effect stemming from the undervaluation of liquid assets by national planners dominates the effect stemming from the undervaluation of illiquid assets, so that wedges in ex-post valuations of assets result in an excessive illiquidity of investment by national planners relative to the constrained efficient allocation (reached in a GRE). Failing to coordinate macro-prudential policy therefore results in an insufficient amount of regulation. Since the global planner's objective is strictly decreasing in k for $k \geq \tilde{k}$, this insufficient amount of regulation results in a weakly lower welfare than in the constrained efficient allocation (strictly lower if $\hat{k} > \frac{1}{1+\kappa r}$). Insufficient provision of regulation also results in more liquidation and higher interest rates during crises.

Without imposing additional structure on the primitives of the model, the relationship between $\hat{k}$ and k^{CE} is ambiguous. As can be guessed from lemma 3, this relationship depends upon two counteracting effects. National planners value both assets less highly than private agents. By hoarding less liquidity than private agents in a competitive equilibrium, national planners benefit from a higher interest rate when their country is intact and lends abroad, but they suffer from a higher interest rate when their country is distressed and borrows from abroad. The interest rate happens to be more sensitive to the lender's supply of liquidity than to borrower's collateral ($|\partial R^{is}/\partial \ell_A| > \partial R^{is}/\partial k_B$), but because of concave utility, goods are more valuable when a country is distressed and borrowing than when it is intact and lending. When risk-aversion is low, the utility benefit associated with the first effect dominates: monopoly rents when the country lends (e.g. in state (i, s) for country A) are more important in utility terms than monopsony rents when the country borrows (in state (s, i)). National planners therefore find it optimal to set aside less liquidity than private agents. When risk-aversion is high, the utility costs associated with the second effect dominate because goods are a lot more valuable in the state where the country is distressed and borrowing. National planners then find it optimal to set aside more liquidity than private agents. This intuition is validated by the following proposition, which provides conditions under which the first of the two effects dominates.

Proposition 3 (NRE vs. CE)**.** *When utility takes the constant relative risk aversion (CRRA) form, there exists some relative risk aversion coefficient $\bar{\gamma} > 1$ such that for all $\gamma < \bar{\gamma}$, the nationally regulated equilibrium features a less liquid investment portfolio than the competitive equilibrium, i.e. $\hat{k} > k^{CE}$ and $\hat{\ell} < \ell^{CE}$.*

Proposition 3 illustrates that the national planners' relatively low valuation of the liquid asset in the state of nature where its country is intact but the foreign country is distressed can result in less liquid investment choices than in the laissez-faire benchmark. Under the conditions of proposition 3, national planners find it optimal to reduce investment in the liquid asset *below* what would be chosen in the free market. This excessive illiquidity results in lower welfare, as measured by the ex-ante expected utility of a representative agent, than both the constrained efficient allocation (achieved by global regulation) *and* the decentralized equilibrium. It also results in more severe financial crises in the form of more liquidation and larger interest rate spikes. This illustrates that uncoordinated regulation can be worse than no regulation at all.

As in the international policy coordination literature, the source of tension here arises from the attempt by national regulators to make use of monopoly and monopsony power in the market for international liquidity during crises. Regulators recognize that less liquidity hoarding ex-ante results in liquidity supply being scarcer in the state of nature where their country will be lending to distressed foreigners. This scarcity induces a higher interest rate, and thus brings about a shift in surplus from foreign borrowers to domestic lenders. Regulators also recognize that less illiquid investment ex-ante results in liquidity demand being smaller in the state of nature where their country will be borrowing from intact foreigners. This smaller demand induces a lower interest rate, and thus to a shift in surplus from foreign lenders to domestic borrowers.

4 International spillovers

The model developed in section 2 can also be used to study the effects of changes in liquidity regulation across borders. Section 4.1 analyzes the impact of changes in regulation in one country on the liquidity profile of foreign investors' portfolio. Section 4.2 looks at how changes in regulation in one country affect the incentive to regulate in the other country, and section 4.3 studies the welfare effects of a unilateral introduction of liquidity regulation by one of the two countries.

4.1 Regulatory spillovers

How do market participants react to changes in liquidity regulation abroad? To address this question, we consider a version of the model of section 2 in which the date 0 choices are set exogenously in country B and made optimally by private agents in country A. A *competitive equilibrium with arbitrary regulation abroad* consists of date 0 decisions (k_A, ℓ_A), date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices and country B regulation (K_B, L_B), the decisions solve the problem in (6) for country A, and the problems in (1) and (2) for both countries; and (b) markets clear.

A tightening of regulation in country B is captured by a marginal decrease in K_B and a corresponding marginal increase in L_B. The following proposition establishes the direction of the effect of a tightening of regulation in country B on private agents' investment choice in country A.

Proposition 4 (Regulatory spillovers)**.** *A tightening of regulation in country B induces private agents in country A to choose a weakly less liquid investment portfolio, i.e.* $\frac{d\ell_A}{dL_B} \leq 0$*. Furthermore, if there is at least one state of nature at date 1 where* $1 < R^\omega < \Delta$*, the a tightening of regulation in country B induces private agents in country A to choose a strictly less liquid investment portfolio, i.e.* $\frac{d\ell_A}{dL_B} < 0$*.*

In order to understand the intuition for these results, it is useful to note that the tightening of regulation in country B affects incentives of country A agents only to the extent that it affects the interest rate in the various states of nature at date 1. The tightening of regulation in country B does not affect private agents' payoffs in country A in states of nature where $R^\omega = 1$ or $R^\omega = \Delta$ since in these cases the interest rate is not locally sensitive to ex-ante choices. However, it results in lower interest rates in states where $1 < R^\omega < \Delta$. In states where country A agents are lending, this reduction in the interest rate lowers the private return on the liquid asset and leaves the return on the illiquid asset unaffected. In states where country A agents are borrowing, the reduction in interest rate increases the private return on the illiquid asset and leaves the return on the liquid asset unaffected. Hence, if it has any international effects, a tightening of regulation in country B decreases the return on the liquid asset and/or increases the return on the illiquid asset for private agents in country A. This naturally leads these agents to rebalance their investment portfolio in favor of illiquid asset.

4.2 Spillovers in incentives to regulate

How are a regulator's incentives affected by a change in liquidity regulation abroad? This section shows that the interaction between national regulators in the model of section 2 can be understood in terms of the strategic substitutability concept of Bulow, Geanakoplos, and Klemperer (1985). In light of the result in section 4.1 that a tightening of regulation abroad induces more risk-taking at home, one could a priori expect that liquidity regulations are strategic complements across countries. In fact, the model delivers precisely the opposite result, as stated in the following proposition.

Proposition 5 (Strategic substitutabilities in national regulations)**.** *In the neighborhood of a symmetric competitive equilibrium, national liquidity regulations are strategic substitutes.*

The intuition for this result is that in the neighborhood of the symmetric competitive equilibrium, the national planners' payoff functions only depend on the other country's investment choices in the states of nature where there is cross-border borrowing and lending, i.e. in states (i, s) and (s, i). As can be seen from (19) and (20), a marginal tightening of regulation in country $-j$, in the form of a marginal decrease in k_{-j} and a corresponding marginal increase in ℓ_{-j}, has the following effects:

- It decreases the interest rate payment $\kappa r k_{-j}$ which country j receives from country $-j$ in the state of nature where intact agents in country j lend to distressed agents in country $-j$. In that state, this leaves the marginal value of the liquid asset unchanged, but leads to an increase in the marginal value of the illiquid asset $Au'\Big(Ak_j + \kappa r k_{-j}\Big)$.

- It increases the loan size ℓ_{-j} which country j receives from country $-j$ in the state of nature where distressed agents in country j borrow from intact agents in country $-j$. In that state, this decreases the marginal value of investing in the liquid asset, $\Delta u'\Big((1-\kappa) r k_j + \Delta(\ell_{-j} + \ell_j)\Big)$, and it decreases the marginal value of investing in the illiquid asset, $(1-\kappa) r u'\Big((1-\kappa) r k_j + \Delta(\ell_{-j} + \ell_j)\Big)$. But because the marginal value of the liquid asset is higher than that of the illiquid asset, the net effect is a decrease in the relative value of the liquid asset.

The combination of these two effects makes the relative attractiveness of a regulation that increases investment in liquid assets a decreasing function of the tightness of regulations in country $-j$. In other words, these two effects imply that the national regulators' actions are strategic substitutes.

4.3 Welfare effects of unilateral regulation

The analysis in section 3 implies that, starting from a competitive equilibrium, the introduction of a regulation requiring agents in both countries to simultaneously increase their holdings of liquid assets marginally is unambiguously welfare improving for all agents.[12] In this context, one might also be interested in the welfare effects of a unilateral introduction of liquidity regulation. In particular, does such an introduction of regulation increase welfare in the regulated country? Does it increase welfare in the unregulated country? These are relevant questions to the extent that their answers matter for the incentives of a national regulator to move first in a world where macroprudential liquidity regulations are absent to start with. This section addresses these issues using the concept of a competitive equilibrium with exogenous regulation abroad, as defined in section 4.1.

The focus is on the effects of the introduction of a small regulation requiring country B agents to increase their holding of liquid assets by a small amount ($dL_B > 0$ with $dK_B = -dL_B$) relative to the symmetric competitive equilibrium level $\ell^{CE} = 1 - k^{CE}$. We consider in turn the welfare effects in the regulated country and in the unregulated country.

[12] This follows from the fact that the global planner's objective is strictly decreasing in k for $k \geq \tilde{k}$ and that $k^{CE} > \tilde{k}$.

Welfare of regulated country

In the neighborhood of the symmetric competitive equilibrium, the ex-ante welfare of agents in the regulated country (country B) is given by

$$\begin{aligned}\Pi^R(L_B,\ell_A) &\equiv \sum_{\omega\in\Omega}\pi^\omega\hat{V}^\omega(1-L_B,1-\ell_A,L_B,\ell_A) \\ &= \pi^{ii}u\Big(A(1-L_B)+L_B\Big)+\pi^{is}u\Big((1-\kappa)r(1-L_B)+\Delta(L_B+\ell_A)\Big) \\ &\quad+\pi^{si}u\Big(A(1-L_B)+\kappa r(1-\ell_A)\Big)+\pi^{ss}u\Big(r(1-L_B)+\Delta L_B\Big).\end{aligned} \tag{25}$$

Therefore, starting from a symmetric competitive equilibrium, the marginal effect on country B's welfare of a tightening of regulation in country B is given by

$$d\Pi^R=\Big[\frac{\partial\Pi^R}{\partial L_B}+\frac{\partial\Pi^R}{\partial\ell_A}\frac{d\ell_A}{dL_B}\Big]dL_B, \tag{26}$$

where the derivatives are evaluated at $(L_B,\ell_A)=(\ell^{CE},\ell^{CE})$. The first term in the brackets in (26) represents the direct effect of a change in liquidity regulation, while the second term reflects its indirect effect, working through the spillovers operating in country A. Without further restriction, it is not possible to determine the sign of $\frac{\partial\Pi^R}{\partial L_B}$, so the direct weflare effect of the tightening of regulation is ambiguous. The tightening of regulation pushes the interest rate down in the two states (i,s) and (s,i) in which there is international borrowing/lending. In state (i,s), this lowering of the interest rate benefits country B's agents who can borrow more cheaply. In state (s,i), it costs country B's agents who lend at a lower rate. Whether the costs are smaller or larger than the benefits is a priori ambiguous. The direction of the indirect welfare effect, however, is unambiguous. Proposition 4 established that in the neighborhood of a symmetric competitive equilibrium a tightening of regulation in country B induces a less liquid date 0 portfolio choice in country A, i.e. that $\frac{d\ell_A}{dL_B}<0$.[13] Furthermore, the proof of lemma 1 established that $(A+\kappa)k^{CE}>[(1-\kappa)r-2\Delta]k^{CE}+2\Delta$, which together with the strict concavity of $u(\cdot)$ implies that $\frac{\partial\Pi^R}{\partial\ell_A}>0$. The indirect effect $\frac{\partial\Pi^R}{\partial\ell_A}\frac{d\ell_A}{dL_B}$ in (26) is therefore strictly negative. A tightening of liquidity regulation in country B induces a less

[13]This follows from the fact that at the symmetric competitive equilibrium date 0 choices, the date 1 interest rate is such that $1<R^\omega<\Delta$ in states (i,s) and (s,i).

liquid investment in country A, and this increases the interest rate in states (i, s) and (s, i). For country B's agents, the losses from an increase in the costs of borrowing from abroad in state (i, s) is only partially offset by the benefits of an increase in the interest rate payment from abroad in state (s, i). A unilateral tightening of regulations in country B thus induces a reduction in the liquidity of investment portfolios abroad, and this feeds back negatively on to country B's welfare.

With additional restrictions on preferences, the model implies that the direct and the indirect effect work in the same direction. This results in an unambiguous overall effect of a tightening of regulation on the regulated country's welfare, as stated in the following proposition.

Proposition 6 (Beggar-thyself unilateral regulation)**.** *Absent initial regulation, when utility takes the CRRA form, there exists some relative risk aversion coefficient* $\bar{\gamma} > 1$ *such that for all* $\gamma < \bar{\gamma}$ *the unilateral introduction of a (small) regulation is welfare reducing for the country introducing the regulation, i.e.* $\frac{d\Pi^R}{dL_B} < 0$.

Under CRRA utility with a low enough relative risk aversion coefficient, the direct effect of a tightening of regulation on the regulated country's welfare is negative, for reasons similar to those underlying the result in proposition 3. In this case, losses from the extra costs of lending at a lower interest rate in state (s, i) are larger than the benefits from cheaper borrowing in state (i, s) for country B's agents. While introducing regulation simultaneously at home and abroad is unambiguously welfare improving for both countries, a unilateral introduction of regulation can be welfare reducing for the regulated country, because of terms-of-trade effects and their associated spillovers. This result is reminiscent of Corsetti and Pesenti (2001), who find that an unexpected unilateral monetary expansion can be welfare reducing for a given country in an environment where, owing to monopolistic distortions in production and nominal rigidities, the same policy would be welfare improving if pursued simultaneously at home and abroad.

Welfare of unregulated country

In the neighborhood of the symmetric competitive equilibrium, the ex-ante welfare of agents in the unregulated country (country A) is given by

$$\Pi^U(\ell_A, L_B) \equiv \Pi^R(L_B, \ell_A),$$

with $\Pi^R(L_B, \ell_A)$ defined in (25). Starting from a symmetric competitive equilibrium, the marginal effect on country A's welfare of a tightening of regulation in country B is therefore given by

$$d\Pi^U = \Big[\frac{\partial \Pi^U}{\partial \ell_A}\frac{d\ell_A}{dL_B} + \frac{\partial \Pi^U}{\partial L_B}\Big] dL_B, \tag{27}$$

where the derivatives are evaluated at $(\ell_A, L_B) = (\ell^{CE}, \ell^{CE})$. The overall effect of a tightening of regulation on the unregulated country's welfare is again given by the sum of an indirect effect and a direct effect. The analysis is facilitated by the fact that, locally, the effect of a small change in a country's investment choice on its own welfare is identical for the regulated and for the unregulated country ($\frac{\partial \Pi^U}{\partial \ell_A}\Big|_{(\ell_A,L_B)=(\ell^{CE},\ell^{CE})} = \frac{\partial \Pi^R}{\partial L_B}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})}$), as is the effect of a small change in a country's investment choice on the other country's welfare ($\frac{\partial \Pi^U}{\partial L_B}\Big|_{(\ell_A,L_B)=(\ell^{CE},\ell^{CE})} = \frac{\partial \Pi^R}{\partial \ell_A}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})}$). The direct effect of a tightening of regulation on the unregulated country's welfare is therefore unambiguously positive, while the direction of the indirect effect is a priori ambiguous. Perhaps paradoxically, it is the unregulated agents' response to the introduction of regulation abroad that may make them worse off than in the absence of any regulation. Without this behavioral response to the introduction of regulation abroad, the unregulated country would necessarily be made better off by the decrease in risk-taking happening abroad. As above, under some restrictions on preferences, the direct and the indirect effects of the introduction of regulation work in the same direction. This results in an unambiguously positive overall effect of a tightening of regulation on the unregulated country's welfare.

Proposition 7 (Prosper-thy neighbor unilateral regulation)**.** *Absent initial regulation, when utility takes the CRRA form, there exists some relative risk aversion coefficient $\bar{\gamma} > 1$ such that for all $\gamma < \bar{\gamma}$ the unilateral introduction of a (small) regulation is welfare improving for* *the unregulated country, i.e.* $\frac{d\Pi^U}{dL_B} > 0$.

With CRRA utility and a low enough relative risk aversion coefficient, the unregulated country's response to the introduction of regulation abroad contributes positively to its welfare. Agents in the unregulated country react by decreasing their investment in liquid assets and increasing their investment in illiquid assets. At the margin, the only impact on their welfare works through the marginal increase in the interest rate in states (i, s) and (s, i) resulting from this portfolio reallocation. The benefits from lending abroad at a higher rate in state (i, s) is larger than the cost of paying a higher rate of foreign loans in state (s, i). A unilateral regulation may therefore generate *prosper thy-neighbor* effects via both the direct and indirect channels.

5 An asset market formulation

This section presents a variant of the model of section 2 in which the intermediation of funds during a crisis occurs via an asset market rather than via a credit market. In this model variant, distressed agents cannot borrow at date 1, but they can sell some of their illiquid assets to intact agents in order to raise funds. When variables are appropriately relabeled, the date 1 equilibrium of this model is isomorphic to the date 1 equilibrium of the credit market model of section 2. All the results of sections 3.1 to 4 derived for the credit market model therefore also apply to the asset market model of the present section.

As in the baseline model of section 2, markets are incomplete. However, instead of being able to share risk indirectly by borrowing and lending on a credit market at date 1, agents are now able to buy and sell the illiquid asset on a spot market at a price q^ω. Intact agents buy x_j^ω, while distressed agents sell $-x_j^\omega$. We assume that sellers cannot sell more than a fraction η of their total capital holdings k_j. This amounts to assuming limited market liquidity for long-term projects, as in Kiyotaki and Moore (2008). We further assume that the long-term projects that are traded need to be shored up by distressed agents before being delivered to a buyer. Buyers of illiquid projects at date 1 therefore receive Ax_j^ω at date 2. All other assumptions of section 2.1 pertaining to preferences, technology and uncertainty are maintained.

At date 1, the value of an intact agent in country j is now given by

$$V_i^\omega(k_j, \ell_j) \equiv \max_{0 \leq x_j^\omega \leq \ell_j / q^\omega} u\Big(Ak_j + Ax_j^\omega + \ell_j - q^\omega x_j^\omega\Big) \tag{28}$$

The agent's date 2 consumption in (28) is given by the sum of the return on its initial holding of illiquid projects Ak_j, the return on the newly acquired illiquid projects Ax_j^ω and the return on the funds invested at date 1 in the storage technology $\ell_j - q^\omega x_j^\omega$. Without loss of generality, we assume that intact agents can only buy and not sell assets on the date 1 spot market. Their capacity to buy is limited by their date 1 liquid resources ℓ_j.

As in the model of section 2, the form of the objective in (28) leads to a simple asset demand schedule for intact agents. For $q^\omega < A$, intact agents exhaust their budget constraint and are willing to buy ℓ_j / q^ω units of the long-term asset. For $q^\omega = A$, they are indifferent between buying any amount between 0 and ℓ_j / A. Finally, for $q^\omega > A$, they do not want to buy any long-term assets. The intact agents' asset demand curve is therefore horizontal at $q^\omega = A$ and slopes downward for $q^\omega < A$, as shown in figure 5.

The date 1 value of a distressed agent in country j is given by

$$V_s^\omega(k_j, \ell_j) \equiv \max_{\theta_j^\omega, x_j^\omega} u\Big(r(1 - \theta_j^\omega)k_j + A(\theta_j^\omega k_j + x_j^\omega) + \ell_j - q^\omega x_j^\omega - \theta_j^\omega k_j\Big) \tag{29}$$

subject to

$$\theta_j^\omega k_j \leq -q^\omega x_j^\omega + \ell_j, \tag{30}$$

$$-x_j^\omega \leq \eta k_j, \tag{31}$$

A distressed agent's date 2 consumption in (29) is given by the sum of the return on the long-term assets that were not shored up, $r(1 - \theta_j^\omega)k_j$, the return on the long-term assets that were shored up but not sold $A(\theta_j^\omega k_j + x_j^\omega)$, and the return on the funds invested at date 1 in the storage technology $\ell_j - q^\omega x_j^\omega - \theta_j^\omega k_j$. (30) is the date 1 budget constraint stating that reinvestment $\theta_j^\omega k_j$ needs to be covered by the sum of ex-ante liquidity hoarding ℓ_j and proceeds of ex-post sale of assets $-q^\omega x_j^\omega$. (31) says that distressed agents cannot resell more than a fraction η of their initial long-term asset holdings k_j.

The form of the objective in (29) again yields a simple form for a distressed agent's asset supply schedule. For $q^\omega < A/\Delta$, distressed agents does not want to sell any assets, since the revenue from a sale is lower than the cost of shoring up the asset. At $q^\omega = A/\Delta$, they are indifferent between selling any amount they can. Finally, for $q^\omega > A/\Delta$, they want to sell as much as possible. The distressed agents' asset supply curve is therefore horizontal at $q^\omega = A/\Delta$ and vertical at ηk_j for $q^\omega > A/\Delta$.

Asset market clearing requires $x_A^\omega + x_B^\omega = 0$. The equilibrium asset price necessarily satisfies $A/\Delta \leq q^\omega \leq A$. The equilibrium is simply given by $q^\omega = A/\Delta$ and $x_A^\omega = x_B^\omega = 0$ in state (i,i), and by $q^\omega = A$ and $x_A^\omega = x_B^\omega = 0$ in state (s,s). When country j is intact and country $-j$ is distressed (i.e. in states (i,s) and (s,i)), under the condition that $A/\Delta < \frac{\ell_j}{\eta k_{-j}} < A$, the equilibrium takes the form displayed in Figure 5, and equating the asset demand ℓ_j/q^ω with the asset supply ηk_{-j} yields an equilibrium asset price of

$$q^\omega = \frac{\ell_j}{\eta k_{-j}}. \tag{32}$$

The equilibrium features *cash-in-the-market pricing*, as in Allen and Gale (1998), in that the asset price depends positively on the amount of liquidity in the hands of the intact country's buyers. A marginal increase in the amount invested ex-ante in the liquid asset by intact agents would push up the asset price by increasing the demand for the asset (shifting the downward sloping part of the demand curve to the right). The equilibrium also exhibits *fire sales* in that the asset price depends negatively on the amount of illiquid assets thrown on the market by the distressed country's sellers. A marginal decrease in the amount of ex-ante illiquid investment by distressed agents would push up the asset price by reducing the supply of the asset (shifting the vertical portion of the supply curve to the left). Naturally, such movements in the asset price have ex-ante welfare implications analogous to movements in the interest rate in the model of section 2.

In the equilibrium just described, the date 1 value function of an intact agent is given by

$$V_i^\omega(k_j, \ell_j) = u\Big(Ak_j + \frac{A}{q^\omega}\ell_j\Big), \tag{33}$$

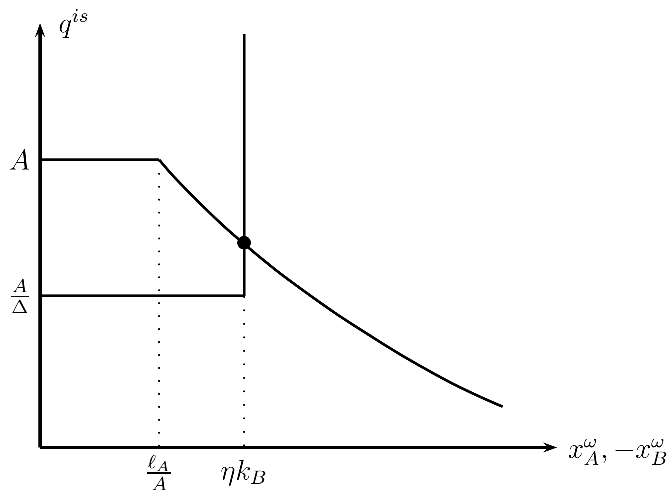

Figure 5: Asset market equilibrium in state (i, s).

and that of a distressed agent is given by

$$V_s^\omega(k_j, \ell_j) = u\Big(r(1-\eta)k_j + \Delta(q^\omega - 1)\eta k_j + \Delta\ell_j\Big). \tag{34}$$

(33) indicates that for an intact agent, illiquid assets yield a return of A, while liquid assets, by allowing the purchase at price q^ω of assets that have an ultimate return of A, yield a return of A/q^ω. The terms in (34) can be interpreted similarly. For a distressed agent, each unit of liquid assets allows shoring up one unit of illiquid assets, yielding a return Δ, whereas each unit of illiquid assets yields a return of r for the share $1-\eta$ that cannot be sold off, and a return of $\Delta(q^\omega - 1)$ for the share η that can be sold off (an asset sale brings an extra $q^\omega - 1$ of date 1 liquidity, whose internal return is Δ).

A remarkable aspect of the asset market equilibrium is its complete isomorphism with the credit market equilibrium of the model of section 2. For $\eta = \frac{\kappa r}{A}$, the equilibrium of the asset market model corresponds exactly to the equilibrium of the credit market model, when prices are redefined according to $R^\omega = \frac{A}{q^\omega}$. Hence, when $\underline{\eta} < \eta < \bar{\eta}$, for $\underline{\eta} = \frac{\underline{\kappa} r}{A}$ and $\bar{\eta} = \frac{\bar{\kappa} r}{A}$, date 0 decisions in a symmetric competitive equilibrium result in the date 1 asset market equilibrium taking the form displayed in Figure 5. The results derived in sections 3.1 to 4 for the credit market model therefore automatically also apply to the asset market model of this section. This illustrates that the coordination problem between national regulatory authorities that is at the core of this paper does not rely on a particular specification of the market allowing distressed entities to raise funds during a crisis. In the asset market version of the model, global regulation calls for more investment in liquid assets and less investment in illiquid assets ex-ante, with the aim of supporting asset prices during financial crises. Marginally higher asset prices in a crisis redistribute wealth from intact buyers to distressed sellers, and thereby achieve a reduction in the cross-sectional wedges between marginal rates of substitution between goods in states (i, s) and (s, i). National planners do not aim at a reduction of these wedges, but rather try to shift surplus in favor of domestic residents, by pushing down the asset price in states where their residents are buying the asset and pushing it up in states where the residents are selling the asset.

6 Conclusion

This paper studies the international coordination problem inherent to the financial stability objective of bank liquidity regulation. Using a multi-country model of liquidity demand, it argues that when incomplete markets and pecuniary externalities are a rational for macro-prudential liquidity regulation, global policy coordination is a necessity, not a luxury. It notably does so by illustrating that regulation without coordination may result in lower welfare than the laissez-faire. The paper also puts forward a mechanism by which liquidity regulation policy spills over across countries. Tighter liquidity regulation at home induces more risk taking abroad via its effect on the price of liquidity in potential future crises. Finally, the analysis indicates that incentives for national authorities to move first in the area of liquidity regulation are weak because of the international public goods properties of liquidity during financial crises.

These findings have implications beyond the framework of the particular model analyzed. They suggest that whenever pecuniary externalities operate across borders, there is a strong case for cooperation in policies whose underlying motivation is to correct such externalities. Hence, they imply that the case for international coordination extends to a large part of the growing research agenda that motivates financial crisis prevention policies from a second best perspective in incomplete markets environments (see reviews in Wagner 2009 and Korinek

2011a). The quantitative relevance of the implied coordination problem is likely to depend on the particular frictions and policies under scrutiny, and its assessment is an important task for future research.

At a more abstract level, the paper also shows that, contrary to common beliefs, market power can increase the magnitude of the distortions induced by systemic externalities. Whether market power attenuates or amplifies these distortions crucially depends on the direction of the relevant pecuniary externalities. The non-trivial interplay between distortions arising from market incompleteness and those due to non-competitive market structures is another fruitful avenue for future research.

References

Acharya, V. V. (2003). Is the international convergence of capital adequacy regulation desirable? *Journal of Finance 58*(6), 2745–2782.

Allen, F. and D. Gale (1998). Optimal financial crises. *Journal of Finance 53*(4), 1245–1284.

Allen, F. and D. Gale (2004). Financial intermediaries and markets. *Econometrica 72*(4), 1023–1061.

Ball, L. and D. Romer (1989). Are prices too sticky? *Quarterly Journal of Economics 104*, 507–524.

Bengui, J. (2011). Systemic risk and inefficient debt maturity. Mimeo, University of Maryland.

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2013). Financial crises and macro-prudential policies. *Journal of International Economics 89*(2), 453–470.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctualtions. *American Economic Review 79*, 1–15.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Bianchi, J. and E. G. Mendoza (2010). Overborrowing, financial crises and 'macro-prudential' policy. Mimeo, University of Maryland.

Blanchard, O. J. and N. Kiyotaki (1987). Monopolistic competition and the effects of aggregate demand. *American Economic Review 77*, 647–666.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy 93*(3), 488–511.

Caballero, R. J. and A. Krishnamurthy (2001). International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics 48*(1), 513–548.

Caballero, R. J. and A. Krishnamurthy (2004). Smoothing sudden stops. *Journal of Economic Theory 116*(1), 104–127.

Castiglionesi, F., F. Feriozzi, and G. Lorenzoni (2010). Financial integration and liquidity crises. Mimeo, Princeton University.

Cooper, R. N. (1985). Economic interdependence and coordination of economic policies. In G. M. Grossman and K. Rogoff (Eds.), *Handbook of International Economics*, Volume 2. Elsevier.

Corsetti, G. and P. Pesenti (2001). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics 116*(2), 421–445.

Costinot, A., G. Lorenzoni, and I. Werning (2012). A theory of capital controls as dynamic terms-of-trade manipulation. Mimeo, MIT.

Dell Arricia, G. and R. Marquez (2006). Competition among regulators and credit market integration. *Journal of Financial Economics 79*, 401–430.

Diamond, D. and P. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Farhi, E., M. Golosov, and A. Tsyvinski (2009). A theory of liquidity and regulation of financial intermediation. *Review of Economic Studies 76*(3), 973–992.

Geanakoplos, J. and H. Polemarchakis (1986). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In W. A. Barnett and K. J. Singleton (Eds.), *Uncertainty, Information and Communication, Essays in Honor of Kenneth J. Arrow*. Cambridge University Press.

Hamada, K. (1974). Alternative exchange rate systems and the interdependence of monetary policies. In R. Aliber (Ed.), *National Monetary Policies and the International Financial System*. University of Chicago Press.

Hamada, K. (1976). A strategic analysis of monetary interdependence. *Journal of Political Economy 84*, 677–700.

Hamada, K. (1979). Macroeconomic strategy and coordination under alternative exchange rates. In R. Dornbush and J. Frenkel (Eds.), *International Economic Policy*. Johns Hopkins University Press.

Hart, O. (1975). On the optimality of equilibrium when the market structure is incomplete. *Journal of Economic Theory 11*, 418–443.

Holmstrom, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), 1–40.

Jeanne, O. and A. Korinek (2011). Managing credit booms and busts: a pigouvian taxation approach. Mimeo, University of Maryland.

Johnson, H. G. (1965). An economic theory of protectionism, tariff bargaining, and the formation of custums union. *Journal of Political Economy 73*(3), 256–283.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*, 211–248.

Kiyotaki, N. and J. Moore (2008). Liquidity, business cycles, and monetary policy. Mimeo, Princeton University.

Korinek, A. (2011a). The new economics of prudential capital controls. *IMF Economic Review*. forthcoming.

Korinek, A. (2011b). Systemic risk-taking: Amplification effects, externalities, and regulatory responses. Mimeo, University of Maryland.

Korinek, A. (2013). Capital controls and currency wars. Mimeo, University of Maryland.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies 75*(3), 809–833.

Morrison, A. D. and L. White (2009). Level playing fields in international financial regulation. *Journal of Finance 64*(3), 1099–1142.

Persson, T. and G. Tabellini (1995). Double-edged incentives: Institutions and policy coordination. In G. M. Grossman and K. Rogoff (Eds.), *Handbook of Devlopment Economics*, Volume 3. Elsevier.

Stiglitz, J. (1982). The inefficiency of stock market equilibrium. *Review of Economic Studies 49*, 241–261.

Wagner, W. (2009). In the quest of systemic externalities: A review of the literature. *CESifo Economic Studies*. forthcoming.

A Proofs

Proof of lemma 1

Without loss of generality, consider state (i, s). For part 1, differentiating (10) (for $j = A$) and (12) with respect to ℓ_A yields

$$\frac{\partial V_i^{is}}{\partial \ell_A} = R^{is} {u'_A}^{is}, \tag{A.1}$$

and

$$\frac{\partial \tilde{V}^{is}}{\partial \ell_A} = R^{is} {u'_A}^{is} + \ell_A \frac{\partial R^{is}}{\partial \ell_A} \left[{u'_A}^{is} - \frac{\Delta}{R^{is}} {u'_B}^{is} \right]. \tag{A.2}$$

where ${u'_A}^{is} \equiv u'\left(Ak_A + R^{is}\ell_A\right)$ and ${u'_B}^{is} \equiv u'\left(rk_B + (\Delta - R^{is})\frac{\kappa r k_B}{R^{is}} + \Delta \ell_B\right)$. Since $R^{is} = \frac{\kappa r k_B}{\ell_A} < \Delta$ and $\frac{\partial R^{is}}{\partial \ell_A} < 0$, it is sufficient to show that ${u'_B}^{is} > {u'_A}^{is}$ in order to prove that $\frac{\partial \tilde{V}^{is}}{\partial \ell_A} > \frac{\partial V_i^{is}}{\partial \ell_A}$. Given the concavity of $u(\cdot)$, this amounts to showing that $Ak_A + R^{is}\ell_A > rk_B + (\Delta - R^{is})\frac{\kappa r k_B}{R^{is}} + \Delta \ell_B$. For a given tuple $(k_A, k_B, \ell_A, \ell_B)$ in region II, we have $\ell_A + \ell_B < k_A$ and $r < 1 < R^{is}$. This implies that $\Delta(\ell_A + \ell_B) + r(\ell_A - \ell_B) < \Delta k_A + 2R^{is}\ell_A$. Adding and subtracting r on the left-hand side, one obtains $\Delta(\ell_A + \ell_B) + rk_B - rk_A < \Delta k_A + 2R^{is}\ell_A$. Since $\Delta \equiv A - r$ and $\ell_A = \frac{\kappa r k_B}{R^{is}}$, this implies that $rk_B + (\Delta - R^{is})\frac{\kappa r k_B}{R^{is}} + \Delta \ell_B < Ak_A + R^{is}\ell_A$. Therefore, ${u'_B}^{is} > {u'_A}^{is}$ and $\frac{\partial \tilde{V}^{is}}{\partial \ell_A} > \frac{\partial V_i^{is}}{\partial \ell_A}$ in region II. For part 2, differentiating (11) (for $j = B$) and (13) with respect to k_B yields

$$\frac{\partial V_s^{is}}{\partial k_B} = \left[r + (\Delta - R^{is}) \frac{\kappa r}{R^{is}} \right] {u'_B}^{is}, \tag{A.3}$$

and

$$\frac{\partial \tilde{V}^{is}}{\partial k_B} = \left[r + (\Delta - R^{is}) \frac{\kappa r}{R^{is}} \right] {u'_B}^{is} + \ell_A \frac{\partial R^{is}}{\partial k_B} \left[{u'_A}^{is} - \frac{\Delta}{R^{is}} {u'_B}^{is} \right]. \tag{A.4}$$

Again, since $R^{is} = \frac{\kappa r k_B}{\ell_A} < \Delta$ and $\frac{\partial R^{is}}{\partial k_B} > 0$, the fact that ${u'_B}^{is} > {u'_A}^{is}$ directly implies that $\frac{\partial \tilde{V}^{is}}{\partial k_B} < \frac{\partial V_s^{is}}{\partial k_B}$ in region II.

Proof of lemma 2

For part 1, differentiating (19) for $j = A$ with respect to ℓ_A yields

$$\frac{\partial \hat{V}_i^{is}}{\partial \ell_A} = R^{is} {u'_A}^{is} + \ell_A \frac{\partial R^{is}}{\partial \ell_A} {u'_A}^{is}. \tag{A.5}$$

Since $R^{is} = \frac{\kappa r k_B}{\ell_A} < \Delta$, $\frac{\partial R^{is}}{\partial \ell_A} < 0$ and ${u'_B}^{is} > 0$, it is clear from (A.2) and (A.5) that $\frac{\partial \tilde{V}_i^{is}}{\partial \ell_A} > \frac{\partial \hat{V}_i^{is}}{\partial \ell_A}$ in region II. For part 2., differentiating (20) for $j = B$ with respect to k_B yields

$$\frac{\partial \hat{V}_s^{is}}{\partial k_B} = \left[r + (\Delta - R^{is})\frac{\kappa r}{R^{is}}\right] {u'_B}^{is} - \ell_A \frac{\partial R^{is}}{\partial k_B}\frac{\Delta}{R^{is}} {u'_B}^{is}. \tag{A.6}$$

Since $R^{is} = \frac{\kappa r k_B}{\ell_A} < \Delta$, $\frac{\partial R^{is}}{\partial k_B} > 0$ and ${u'_A}^{is} > 0$, it is clear from (A.4) and (A.6) that $\frac{\partial \tilde{V}_i^{is}}{\partial k_B} > \frac{\partial \hat{V}_i^{is}}{\partial k_B}$ in region II.

Proof of lemma 3

For part 1, we simply observe from (A.5) and (A.1) that $\frac{\partial R^{is}}{\partial \ell_A} < 0$ implies $\frac{\partial \hat{V}_i^{is}}{\partial \ell_A} < \frac{\partial V_i^{is}}{\partial \ell_A}$ in region II.

For part 2, we similarly observe from (A.6) and (A.3) that $\frac{\partial R^{is}}{\partial k_B} > 0$ implies that $\frac{\partial \hat{V}_i^{is}}{\partial k_B} < \frac{\partial V_i^{is}}{\partial k_B}$ in region II.

Proof of proposition 1

We start by defining the two functions characterizing a symmetric competitive equilibrium and a symmetric globally regulated equilibrium, respectively,

$$\begin{aligned} g_{II}(k) \equiv\ & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big) + \pi^{is}\Big(A - \frac{\kappa r k}{1-k}\Big)u'\Big((A+\kappa r)k\Big) \\ & + \pi^{si}\Big[r + \Big(\Delta - \frac{\kappa r k}{1-k}\Big)\frac{\kappa r}{\frac{\kappa r k}{1-k}} - \Delta\Big]u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) + \pi^{ss}(r-\Delta)u'\Big(rk + \Delta(1-k)\Big), \end{aligned} \tag{A.7}$$

and

$$\begin{aligned} \tilde{g}_{II}(k) \equiv\ & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big) + \pi^{is}\Big[Au'\Big((A+\kappa r)k\Big) - \Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big)\Big] \\ & + \pi^{si}\Big[(1-\kappa)ru'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) + \kappa r u'\Big((A+\kappa r)k\Big) - \Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big)\Big] \\ & + \pi^{ss}(r-\Delta)u'\Big(rk + \Delta(1-k)\Big). \end{aligned} \tag{A.8}$$

Under assumption 3, the value of k in a symmetric competitive equilibrium, k^{CE}, falls in the interior of region II, $k^{CE} \in \Big(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big)$, and is implicitly given by $g_{II}(k^{CE}) = 0$. A necessary condition for a globally regulated equilibrium is that $\tilde{g}_{II}(\tilde{k}) = 0$ if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{\Delta}{\Delta+\kappa r}$, $\tilde{g}_{II}(\tilde{k}) \leq 0$ if $\tilde{k} = \frac{1}{1+\kappa r}$ and $\tilde{g}_{II}(\tilde{k}) \geq 0$ if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $g'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\tilde{g}_{II}(k) < g_{II}(k)$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, then either $\tilde{k} \in \Big[\frac{1}{1+\kappa r}, k^{CE}\Big)$ with $\tilde{g}_{II}(\tilde{k}) = 0$, or $\tilde{k} = \frac{1}{1+\kappa r} < k^{CE}$ with $\tilde{g}_{II}(\tilde{k}) < 0$. It follows that showing that $g'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\tilde{g}_{II}(k) < g_{II}(k)$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$ is sufficient to prove that $\tilde{k} < k^{CE}$.

The derivative of $g_{II}(\cdot)$ is given by:

$$\begin{aligned} g'_{II}(k) \equiv\; & \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) + \pi^{ss}(r-\Delta)^2 u''\Big(rk+\Delta(1-k)\Big) \\ & +\pi^{is}\Big[-\frac{\kappa r(1-k)+\kappa rk}{(1-k)^2}u'\Big((A+\kappa r)k\Big) + \Big(A - \frac{\kappa rk}{1-k}\Big)(A+\kappa r)u''\Big((A+\kappa r)k\Big)\Big] \\ & +\pi^{si}\Big\{\Big[r + \Big(\Delta - \frac{\kappa rk}{1-k}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-k}} - \Delta\Big]\Big((1-\kappa)r - 2\Delta\Big)u''\Big((1-\kappa)rk + 2\Delta(1-k)\Big) \\ & -(\Delta+\kappa r)u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big)\Big\}. \end{aligned}$$

Each single term of $g'_{II}(k)$ is strictly negative for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $g'_{II}(k) < 0$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$.

Similarly, the derivative of $\tilde{g}_{II}(\cdot)$ is given by:

$$\begin{aligned} \tilde{g}'_{II}(k) \equiv\; & \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) + \pi^{ss}(r-\Delta)^2 u''\Big(rk+\Delta(1-k)\Big) \\ & +\pi^{is}\Big[A(A+\kappa r)u''\Big((A+\kappa r)k\Big) - \Delta[(1-\kappa)r - 2\Delta]u''\Big((1-\kappa)rk + 2\Delta(1-k)\Big)\Big] \\ & +\pi^{si}\Big[[(1-\kappa)r - \Delta][(1-\kappa)r - 2\Delta]u''\Big((1-\kappa)rk + 2\Delta(1-k)\Big) + \kappa r(A+\kappa r)u''\Big((A+\kappa r)k\Big)\Big]. \end{aligned}$$

Every single term is strictly negative for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, sot $\tilde{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$.

It remains to show that $\tilde{g}_{II}(k) < g_{II}(k)$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. Defining $\tilde{\Phi}(k) \equiv \tilde{g}_{II}(k) - g_{II}(k)$, we have

$$\begin{aligned} \tilde{\Phi}(k) =\; & -\pi^{is}\Big[\Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) - \frac{\kappa rk}{1-k}u'\Big((A+\kappa r)k\Big)\Big] \\ & -\pi^{is}\kappa r\Big[u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) - u'\Big((A+\kappa r)k\Big)\Big]. \end{aligned}$$

Lemma 1 established that the terms in brackets are positive for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, implying that $\tilde{\Phi}(k) < 0$ for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. It follows that $\tilde{k} < k^{CE}$, and therefore that $\tilde{\ell} > \ell^{CE}$.

Proof of proposition 2

We start by defining the function characterizing a symmetric nationally regulated equilibrium

$$\begin{aligned} \hat{g}_{II}(k) \equiv\; & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big) + \pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big) + \pi^{is}Au'\Big((A+\kappa r)k\Big) \\ & +\pi^{si}\Big[(1-\kappa)ru'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) - \Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big)\Big] \end{aligned} \tag{A.9}$$

Under assumption 3, the value of k in a symmetric competitive equilibrium, k^{CE}, falls in the interior of region II, $k^{CE} \in \left(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right)$, and is implicitly given by $g_{II}(k^{CE}) = 0$ (with $g_{II}(\cdot)$ defined in A.7). A necessary condition for a globally regulated equilibrium is that $\tilde{g}_{II}(\tilde{k}) = 0$ (with $\tilde{g}_{II}(\cdot)$ defined in A.8) if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{\Delta}{\Delta+\kappa r}$, $\tilde{g}_{II}(\tilde{k}) \leq 0$ if $\tilde{k} = \frac{1}{1+\kappa r}$ and $\tilde{g}_{II}(\tilde{k}) \geq 0$ if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$. Similarly, a necessary condition for a nationally regulated equilibrium is that $\hat{g}_{II}(\hat{k}) = 0$ if $\frac{1}{1+r\kappa} < \hat{k} < \frac{\Delta}{\Delta+\kappa r}$, $\hat{g}_{II}(\hat{k}) \leq 0$ if $\hat{k} = \frac{1}{1+\kappa r}$ and $\hat{g}_{II}(\hat{k}) \geq 0$ if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $\hat{g}'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > \tilde{g}_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+\kappa r}\right]$, then we can be a priori in either of the following three situations: (1) $\hat{k} = \tilde{k} = \frac{1}{1+\kappa r}$, with $\hat{g}_{II}(\hat{k}) \leq 0$, $\tilde{g}_{II}(\tilde{k}) < 0$; (2) $\tilde{k} = \frac{1}{1+\kappa r} < \hat{k}$, with $\hat{g}_{II}(\hat{k}) \geq 0$, $\tilde{g}_{II}(\tilde{k}) \leq 0$; (3) $\frac{1}{1+\kappa r} < \tilde{k} < k^{CE}$ and $\tilde{k} < \hat{k}$ with $\hat{g}_{II}(\hat{k}) \geq 0$, $\tilde{g}_{II}(\tilde{k}) = 0$. It follows that showing that $\hat{g}'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > \tilde{g}_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ is sufficient to prove that $\hat{k} \geq \tilde{k}$. Moreover, if in addition $\hat{k} > \frac{1}{1+\kappa r}$, then $\hat{k} > \tilde{k}$.

The fact that $\tilde{g}'_{II}(k)$ for $k \in \left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ has been established as part of proposition 1. The derivative of $\hat{g}_{II}(\cdot)$ is given by:

$$\begin{aligned}\hat{g}'_{II}(k) \quad \equiv \quad & \pi^{ii}(A-1)^2u''\Big((A-1)k+1\Big) + \pi^{ss}(r-\Delta)^2u''\Big(rk+\Delta(1-k)\Big) + \pi^{is}A(A+\kappa r)u''\Big((A+\kappa r)k\Big) \\ & +\pi^{si}[(1-\kappa)r-\Delta][(1-\kappa)r-2\Delta]u''\Big((1-\kappa)rk+2\Delta(1-k)\Big)\end{aligned}$$

Every single term is strictly negative for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $\hat{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$.

Now, defining $\check{\Phi}(k) \equiv \hat{g}_{II}(k) - \tilde{g}_{II}(k)$, we have

$$\check{\Phi}(k) = \pi^{is}\Big[\Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) - \kappa r u'\Big((A+\kappa r)k\Big)\Big] > 0$$

The term in bracket is positive for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $\check{\Phi}(k) > 0$ for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. It follows that: (1) $\hat{k} \geq \tilde{k}$ and $\hat{\ell} \leq \tilde{\ell}$; (2) if $\hat{k} > \frac{1}{1+\kappa r}$ then $\hat{k} > \tilde{k}$ and $\hat{\ell} < \tilde{\ell}$.

Proof of proposition 3

Under assumption 3, $k^{CE} \in \left(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right)$ and $g_{II}(k^{CE}) = 0$ (with $g_{II}(\cdot)$ defined in (A.7)). A necessary condition for a nationally regulated equilibrium is that $\hat{g}_{II}(\hat{k}) = 0$ (with $\hat{g}_{II}(\cdot)$ defined in (A.9)) if $\frac{1}{1+\kappa r} < \hat{k} < \frac{\Delta}{\Delta+\kappa r}$, $\hat{g}_{II}(\hat{k}) \leq 0$ if $\hat{k} = \frac{1}{1+\kappa r}$ and $\hat{g}_{II}(\hat{k}) \geq 0$ if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $g'_{II}(k) < 0$, $\hat{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > g_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$, then either $\hat{k} \in \left(k^{CE}, \frac{\Delta}{\Delta+\kappa r}\right]$ with $\hat{g}_{II}(\hat{k}) = 0$, or $\hat{k} = \frac{\Delta}{\Delta+\kappa r} > k^{CE}$ with $\hat{g}_{II}(\hat{k}) \geq 0$. The fact that $g'_{II}(k) < 0$ and $\hat{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ has already been established in the proofs of propositions 1 and 2. It follows that showing that $\hat{g}_{II}(k) > g_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ is sufficient to prove that $\hat{k} > k^{CE}$.

Defining $\hat{\Phi}(k) \equiv \hat{g}_{II}(k) - g_{II}(k)$, we have

$$\hat{\Phi}(k) = \frac{\kappa rk}{1-k}u'\Big((A+\kappa r)k\Big) - \Delta\frac{\kappa r(1-k)}{\kappa rk}u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) \tag{A.10}$$

We now show that for the class of CRRA utility functions $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, there exists a $\bar{\gamma} > 1$ such that for all $\gamma < \bar{\gamma}$ we have $\hat{\Phi}(k) > 0$ (and therefore $\hat{g}_{II}(k) > g_{II}(k)$) over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. We start by noting that for $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, (A.10) is given by

$$\hat{\Phi}(k) = \frac{\kappa rk}{1-k}\Big((A+\kappa r)k\Big)^{-\gamma} - \Delta\frac{\kappa r(1-k)}{\kappa rk}\Big((1-\kappa)rk + 2\Delta(1-k)\Big)^{-\gamma}. \tag{A.11}$$

When $\gamma = 1$, $\hat{\Phi}(k) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ if and only if the transformed function

$$\hat{\Phi}_T(k) \equiv [(A+\kappa r)(1-k)][(1-\kappa)rk^2 + 2\Delta(1-k)k]\hat{\Phi}(k) > 0$$

over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. Conveniently, $\hat{\Phi}_T(k)$ is quadratic:

$$\hat{\Phi}_T(k) = (1-\kappa)\kappa r^2k^2 + 2\Delta\kappa r(1-k)k - \Delta(A+\kappa r)(1-k)^2,$$

with a derivative given by

$$\hat{\Phi}'_T(k) = 2(1-\kappa)\kappa r^2k + 2\Delta\kappa r(1-k) - 2\Delta\kappa rk + 2\Delta(A+\kappa r)(1-k).$$

When evaluated at the bounds $\frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r}$, both $\hat{\Phi}_T(k)$ and $\hat{\Phi}'_T(k)$ are strictly positive under assumption 1 that $A \leq \frac{3}{2}$:

$$\begin{aligned}
\hat{\Phi}_T(\frac{1}{1+r\kappa}) &= \frac{\kappa r^2}{(1+\kappa r)^2}\Big[(1-\kappa) + \Delta\kappa[2-(A+\kappa r)\Big] > 0 \\
\hat{\Phi}_T(\frac{\Delta}{\Delta+r\kappa}) &= \frac{\Delta\kappa r^2}{(\Delta+\kappa r)^2}\Big[\Delta(1-\kappa) + \Delta\kappa[2\Delta-(A+\kappa r)]\Big] > 0 \\
\hat{\Phi}'_T(\frac{1}{1+r\kappa}) &= \frac{2\kappa r}{1+\kappa r}\Big[(1-\kappa)r + \Delta[2\kappa r + (A-1)]\Big] > 0 \\
\hat{\Phi}'_T(\frac{\Delta}{\Delta+r\kappa}) &= \frac{2\Delta\kappa r}{\Delta+\kappa r}\Big[(1-\kappa)r + 2\kappa r + (A-\Delta)\Big] > 0
\end{aligned}$$

Due to linearity, $\hat{\Phi}'_T(\cdot) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. This, together with the strict positiveness of $\hat{\Phi}_T(\frac{1}{1+r\kappa})$ and $\hat{\Phi}_T(\frac{\Delta}{\Delta+r\kappa})$, guarantees that $\hat{\Phi}_T(\cdot) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. In turn, this implies that

$\hat{\Phi}(\cdot) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ for $\gamma = 1$. By continuity, it must be that $\hat{\Phi}(\cdot) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ for $\gamma = 1 + \varepsilon$, for some small enough $\varepsilon > 0$. Now, note that for an arbitrary γ, $\hat{\Phi}(\cdot) > 0$ if and only if

$$\left(\frac{(1-\kappa)rk + 2\Delta(1-k)}{(A+\kappa r)k}\right)^{\gamma} > \frac{\Delta}{\kappa r}\left(\frac{1-k}{k}\right)^{2}. \tag{A.12}$$

Since $\frac{(1-\kappa)rk+2\Delta(1-k)}{(A+\kappa r)k} < 1$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ (see lemma 1), if (A.12) holds over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ for some $\bar{\gamma}$, then it also holds over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ for any $\gamma < \bar{\gamma}$, and therefore $\hat{\Phi}(k) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ for $\gamma < \bar{\gamma}$. Thus, we conclude that for all $\gamma < 1 + \varepsilon$, we have $\hat{k} > k^{CE}$ and $\hat{\ell} < \ell^{CE}$.

Proof of proposition 4

Let us define $W(\ell_A, R^{\omega}) \equiv V(1-\ell_A, \ell_A, R^{\omega})$. Country A agent's private optimal date 0 choice is then characterized by $\sum_{\omega\in\Omega} \pi^{\omega} \frac{\partial W(\ell_A, R^{\omega})}{\partial \ell_A} = 0$. According to the implicit function theorem, the spillover effect is given by

$$\frac{d\ell_A}{dL_B} = -\frac{\sum_{\omega\in\Omega} \pi^{\omega} \frac{\partial^2 W(\ell_A, R^{\omega})}{\partial \ell_A \partial R^{\omega}} \frac{\partial R^{\omega}}{\partial L_B}}{\sum_{\omega\in\Omega} \pi^{\omega} \frac{\partial^2 W(\ell_A, R^{\omega})}{\partial \ell_A^2}} \tag{A.13}$$

The negativity of the denominator of (A.13) follows directly from the agent's second order condition. We now establish that each term of the sum in the numerator of (A.13) is non-positive, and that therefore the numerator is non-positive as well. Note that the date 1 interest rate satisfies $1 \leq R^{\omega} \leq \Delta$. In what follows, we distinguish corner cases ($R^{\omega} = 1$ or $R^{\omega} = \Delta$) from interior cases ($1 < R^{\omega} < \Delta$). First, consider the corner cases. In these cases, we have $\frac{\partial R^{\omega}}{\partial L_B} = 0$ and therefore $\pi^{\omega} \frac{\partial^2 W(\ell_A, R^{\omega})}{\partial \ell_A \partial R^{\omega}} \frac{\partial R^{\omega}}{\partial L_B} = 0$ (since $\frac{\partial^2 W(\ell_A, R^{\omega})}{\partial \ell_A \partial R^{\omega}}$ is always finite). Second, consider the interior cases. In these cases, if the country A agent is a lender, then $d_A^{\omega} = -\ell_A$ and $R^{\omega} = \frac{\kappa r(1-L_B)}{L_A}$, and if he is a borrower, then $d_A^{\omega} = \frac{\kappa r(1-\ell_A)}{R^{\omega}}$ and $R^{\omega} = \frac{\kappa r(1-L_A)}{L_B}$. Hence, when the country A agent is a lender, its date 1 value function is given by $W(\ell_A, R^{\omega}) = u(A(1-\ell_A) + \ell_A R^{\omega})$. The relevant derivatives are therefore given by $\frac{\partial R^{\omega}}{\partial L_B} = -\frac{\kappa r}{L_A} < 0$ and

$$\frac{\partial^2 W(\ell_A, R^{\omega})}{\partial \ell_A \partial R^{\omega}} = u'(A(1-\ell_A) + \ell_A R^{\omega}) - (A - R^{\omega})\ell_A u''(A(1-\ell_A) + \ell_A R^{\omega}) > 0$$

When the country A agent is a borrower, its date 1 value function is given by $W(\ell_A, R^{\omega}) = u\left(\left[r + (\Delta - R^{\omega})\frac{\kappa r}{R^{\omega}}\right](1-\ell_A) + \Delta \ell_A\right)$. The relevant derivatives are therefore given by $\frac{\partial R^{\omega}}{\partial L_B} =$

$-\frac{\kappa r(1-L_A)}{(L_B)^2} < 0$ and

$$\frac{\partial^2 W\left(\ell_A, R^\omega\right)}{\partial \ell_A \partial R^\omega} = \left[\frac{\kappa r}{R^\omega} + \frac{\left(\Delta - R^\omega\right)\kappa r}{\left(R^\omega\right)^2}\right]\left[u'\left(c_A^\omega\right) - \left\{\Delta - \left[r + \left(\Delta - R^\omega\right)\frac{\kappa r}{R^\omega}\right]\right\}(1-\ell_A)u''\left(c_A^\omega\right)\right] > 0$$

with $c_A^\omega = \left[r + \left(\Delta - R^\omega\right)\frac{\kappa r}{R^\omega}\right](1-\ell_A) + \Delta \ell_A$. Hence, whether the country A agent is a lender or a borrower, when $1 < R^\omega < \Delta$ we have $\frac{\partial^2 W(\ell_A, R^\omega)}{\partial \ell_A \partial R^\omega} > 0$ and $\frac{\partial R^\omega}{\partial L_B} < 0$, and therefore $\pi^\omega \frac{\partial^2 W(\ell_A, R^\omega)}{\partial \ell_A \partial R^\omega}\frac{\partial R^\omega}{\partial L_B} < 0$.

For every ω, we have $\pi^\omega \frac{\partial^2 W(\ell_A, R^\omega)}{\partial \ell_A \partial R^\omega}\frac{\partial R^\omega}{\partial L_B} \leq 0$, with $<$ if $1 < R^\omega < \Delta$. It follows that $\sum_{\omega \in \Omega} \pi^\omega \frac{\partial^2 W(\ell_A, R^\omega)}{\partial \ell_A \partial R^\omega}\frac{\partial R^\omega}{\partial L_B} \leq 0$, with $<$ if there is at least one ω for which $1 < R^\omega < \Delta$. According to (A.13), this implies that $\frac{d\ell_A}{dL_B} \leq 0$, with $<$ if there is at least one ω for which $1 < R^\omega < \Delta$.

Proof of proposition 5

In the neighborhood of the symmetric competitive equilibrium, country A's national planner payoff can be written as

$$\begin{aligned} \Pi(\ell_A, \ell_B) &= \pi^{ii} u\Big(A(1-\ell_A) + \ell_A\Big) + \pi^{is} u\Big(A(1-\ell_A) + \kappa r(1-\ell_B)\Big) \\ &\quad + \pi^{si} u\Big((1-\kappa) r(1-\ell_A) + \Delta(\ell_A + \ell_B)\Big) + \pi^{ss} u\Big(r(1-\ell_A) + \Delta \ell_A\Big). \end{aligned} \tag{A.14}$$

The cross derivative is given by

$$\frac{\partial^2 \Pi}{\partial \ell_A \partial \ell_B} = \pi^{is} \kappa r A u''\Big(A(1-\ell_A) + \kappa r(1-\ell_B)\Big) + \pi^{si}\Delta[\Delta - (1-\kappa) r] u''\Big((1-\kappa) r(1-\ell_A) + \Delta(\ell_A + \ell_B)\Big),$$

which is unambiguously negative. It follows that the regulators' actions are strategic substitutes.

Proof of proposition 6

The partial derivatives of $\Pi^R(L_B, \ell_A)$, evaluated at (ℓ^{CE}, ℓ^{CE}), are given by

$$\begin{aligned} \frac{\partial \Pi^R}{\partial L_B} &= -\pi^{ii}(A-1)u'\Big(A - (A-1)\ell^{CE}\Big) - \pi^{is}[(1-\kappa)r - \Delta]u'\Big((1-\kappa)r - [(1-\kappa)r - 2\Delta]\ell^{CE}\Big) \\ &\quad - \pi^{si} A u'\Big((A + \kappa r)(1-\ell^{CE})\Big) - \pi^{ss}(r - \Delta)u'\Big(r + (\Delta - r)\ell^{CE}\Big) \end{aligned} \tag{A.15}$$

and

$$\frac{\partial \Pi^R}{\partial \ell_A} = \pi^{is}\Delta u'\Big((1-\kappa)r - [(1-\kappa)r - 2\Delta]\ell^{CE}\Big) - \pi^{si}\kappa r u'\Big((A+\kappa r)(1-\ell^{CE})\Big). \tag{A.16}$$

Using the first-order condition of the symmetric competitive equilibrium, $g_{II}(1-\ell^{CE}) = 0$, we can write (A.15) as $\frac{\partial \Pi^R}{\partial L_B} = -\pi^{is}\hat{\Phi}(1-\ell^{CE})$, where $\hat{\Phi}(k) \equiv \hat{g}_{II}(k) - g_{II}(k)$, for $g_{II}(\cdot)$ and $\hat{g}_{II}(\cdot)$ defined in (A.7) and (A.9), respectively. The proof of proposition 3 established that when the utility function takes the CRRA form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, there exists a $\bar{\gamma} > 1$ such that $\hat{\Phi}(k) > 0$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$ for all $\gamma < \bar{\gamma}$. Since $k^{CE} \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, it follows that there exists a $\bar{\gamma} > 1$ such that $\hat{\Phi}(1-\ell^{CE}) > 0$ and therefore $\frac{\partial \Pi^R}{\partial L_B}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} < 0$ for all $\gamma < \bar{\gamma}$.

Next, the proof of lemma 1 established that $(A+\kappa)(1-\ell^{CE}) > [(1-\kappa)r - 2\Delta](1-\ell^{CE}) + 2\Delta$. Together with the strict concavity of $u(\cdot)$, this implies that $\frac{\partial \Pi^R}{\partial \ell_A}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} > 0$.

Finally, proposition 4 established that $\frac{d\ell_A}{dL_B}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} < 0$. Hence, there exists a $\bar{\gamma} > 1$ such that for all $\gamma < \bar{\gamma}$ the direct and the indirect effects of a change in country B's regulation on country B's welfare work in the same direction:

$$\frac{d\Pi^R}{dL_B}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} = \left[\frac{\partial \Pi^R}{\partial L_B} + \frac{\partial \Pi^R}{\partial \ell_A}\frac{d\ell_A}{dL_B}\right]_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} < 0.$$

Proof of proposition 7

First, observe that at $(\ell_A, L_B) = (\ell^{CE}, \ell^{CE})$, $\frac{\partial \Pi^U}{\partial \ell_A} = \frac{\partial \Pi^R}{\partial L_B}$ and $\frac{\partial \Pi^U}{\partial L_B} = \frac{\partial \Pi^R}{\partial \ell_A}$. Then, note that the proof of proposition 6 established the following results: (1) when the utility function takes the CRRA form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, there exists a $\bar{\gamma} > 1$ such that $\frac{\partial \Pi^R}{\partial L_B}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} < 0$ for all $\gamma < \bar{\gamma}$; (2) $\frac{\partial \Pi^R}{\partial \ell_A}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} > 0$; (3) $\frac{d\ell_A}{dL_B}\Big|_{(L_B,\ell_A)=(\ell^{CE},\ell^{CE})} < 0$. From (1), it follows that there exists a $\bar{\gamma} > 1$ such that $\frac{\partial \Pi^U}{\partial \ell_A}\Big|_{(\ell_A,L_B)=(\ell^{CE},\ell^{CE})} < 0$ for all $\gamma < \bar{\gamma}$. From (2), it follows that $\frac{\partial \Pi^U}{\partial L_B}\Big|_{(\ell_A,L_B)=(\ell^{CE},\ell^{CE})} > 0$.

Hence, there exists a $\bar{\gamma} > 1$ such that for all $\gamma < \bar{\gamma}$ the direct and indirect effects of a change in country B's regulation on country A's welfare work in the same direction:

$$\frac{d\Pi^U}{dL_B}\Big|_{(\ell_A,L_B)=(\ell^{CE},\ell^{CE})} = \left[\frac{\partial \Pi^U}{\partial \ell_A}\frac{d\ell_A}{dL_B} + \frac{\partial \Pi^U}{\partial L_B}\right]_{(\ell_A,L_B)=(\ell^{CE},\ell^{CE})} > 0.$$