# Stagflation and Topsy-Turvy Capital Flows*

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**Abstract**

Should we expect free capital mobility to facilitate macroeconomic adjustment in a stagflation context? We argue that according to standard models used to analyze stabilization policy, the answer is negative. These models feature a previously undocumented macroeconomic externality by which capital flows undermine stabilization policy when the latter faces an output-inflation trade-off. The externality operates via the economy's supply side: capital inflows raises domestic wages and cause unwelcome upward pressure on marginal costs in high-inflation countries. Yet, market forces are likely to generate inflows into these countries. A constrained efficient regime would instead require net outflows, suggesting topsy-turvy capital flows following supply shocks.

**Keywords:** Stagflation, current account adjustment, macroeconomic externalities, stabilization policy, capital flow management

**JEL Classifications:** E32, E44, E52, F32, F41, F42

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# 1 Introduction

One of the most striking macroeconomic developments of the ongoing recovery is the recent pick up in inflation, shown in the left panel of Figure 1 for G7 economies. To prevent inflation from settling at high levels, major central banks have engaged in their most aggressive tightening cycle in decades, as shown in the figure's right panel. This rapid increase in interest rates across advanced economies is raising the spectre of a new "taper tantrum," by which global investors' search for yield leads to large capital outflows from emerging economies. Should we be concerned about these capital outflows, and the mirroring capital inflows into economies with rapidly tightening policy stances, being excessive? Are the rising odds of stagflation critical for this assessment? Over the past twenty years, a large body of research in macroeconomic theory has increasingly pointed to imperfections in financial, goods and labor markets as possible causes of excessive capital flows (e.g., Bianchi 2011 and Schmitt-Grohe and Uribe 2016). Yet, perhaps because adverse supply shocks have been off policymakers' radar for much of this period, the literature has largely ignored the role of the trade-off between stabilizing inflation and limiting economic activity, which appears central in the current context. Our goal in this paper is to squarely focus on this issue.

We point to a previously undocumented macroeconomic externality associated with capital flows and operating through the economy's supply-side: By propping up consumption, capital inflows shift households' labor supply schedule up and, when the trade elasticity is greater than the degree of home bias, raise domestic firms' marginal costs.[1] When the economy operates at potential and inflation is perfectly stabilized, this externality does not cause any inefficiency. But when the economy operates below potential as a result of the central bank's attempt to fight off a cost-push shock (i.e., in a stagflation scenario), the rise in marginal costs worsens the policy trade-off: To stabilize inflation at a given level, the central bank needs to engineer a more severe recession. In this context, the macroeconomic externality generates first-order welfare effects and creates a wedge between the privately and socially optimal levels of external borrowing.

We formalize this insight in a simple two-country general equilibrium model with nominal rigidities, whose building blocks form the backbone of more complex dynamic stochastic general equilibrium (DSGE) models used by most central banks for policy

[1] This condition on the trade elasticity is weaker than the well-known Marshall-Lerner condition, which states that the sum of a country's export and import demand elasticities, commonly defined as the trade elasticity, is greater than one. When this condition is satisfied, an exchange rate depreciation improves the trade balance.

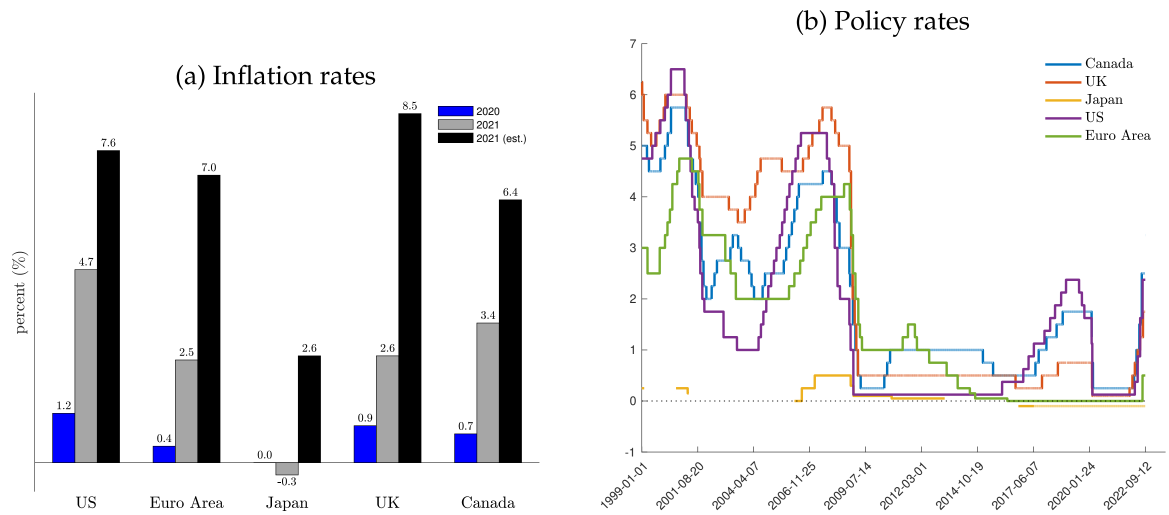


Figure 1: Inflation and policy rates in G7 countries.

*Note:* Inflation rates (left) show CPI inflation. Bloomberg consensus forecasts were used to compute 2022 estimates. Policy rates (right) are daily data from the BIS. Canada: Central Bank target for the overnight rate; UK: repo rate prior to August 3, 2006, and official bank rate from August 3, 2006 onwards; Japan: see Appendix; US: midpoint of the Federal Reserve target rate; Euro Area: main refinancing operations, minimum bid, prior to October 15, 2008, and official central bank liquidity providing, main refinancing operations, fixed rate, from October 15, 2008 onwards.

analysis. Assuming that labor supply, expenditure allocation and price-setting decisions are made by individual households and firms, we ask what level of external saving or borrowing a planner would choose.

We find that under free capital mobility, capital tends to flow excessively toward the country where the degree of stringency of the output-inflation trade-off is the highest, due to the externality described above.[2] In our model, firms' marginal costs can be decomposed into a pure labor cost component, depending on the real wage measured in terms of the economy's consumption basket, and an adjustment term accounting for the economy's relative purchasing power. For a given output gap, a marginal increase in external borrowing raises domestic spending, shifting up the labor supply schedule and causing a rise in the equilibrium real wage. When the model features home bias in consumption, the increase in domestic spending also appreciates the terms of trade, which raises the purchasing power of domestic firms and attenuates the rise in their marginal costs. If the trade elasticity is larger than the degree of home bias, the direct effect on labor costs outweighs the latter effect on purchasing power, hence leading to an overall increase

[2] In our model, such cross-country differences in the stringency of output-inflation trade-offs are the result of asymmetric cost-push shocks, but more generally, they could arise from any structural asymmetry across countries.

in firms' marginal costs.

Intuitively, when monetary policy already faces an unfavorable trade-off and adopts a particularly tight stance to limit domestic inflation at the expense of undesirably low economic activity, the upward pressure on domestic marginal costs caused by additional capital inflows make monetary policy's job even harder. Either the central bank lets the rise in marginal costs translate into higher domestic inflation, or it is forced to depress economic activity further to achieve a given stabilization of inflation. Either way, the economy is worse off, and this adverse side effect of external borrowing is not adequately signaled by its price in an unregulated market.

The externality we point to does not simply lead to inefficiencies at the margin. Indeed, it can be powerful enough to reverse the direction of capital flows in response to cost-push shocks. While a free capital mobility regime is likely to feature capital inflows into the region with the most depressed activity, an optimal capital flow management regime always prescribes outflows from that region. Our analysis hence suggests that ostensibly wrong price signals in international financial markets can lead to *tospy-turvy* capital flows during a stagflation episode.

Our externality resembles those stressed by two branches of the recent literature in monetary and international macroeconomics. In the first one, elegantly exposed in general terms by Farhi and Werning (2016), privately optimal financial choices differ from socially optimal ones due to aggregate demand externalities in economies with nominal rigidities.[3] In the second one, following Caballero and Krishnamurthy (2001), pecuniary externalities generate inefficiencies in incomplete markets environments.[4] Our externality occurs in a setting of complete financial markets and aggregate demand imbalances, like aggregate demand externalities, but works through prices, like pecuniary externalities. To clarify that our externality works through market prices rather than quantity adjustments, we use a special case of our model, featuring no home bias in consumption. In this case, marginal costs only respond to external borrowing to the extent that the latter affects real wages via a wealth effect on labor supply. Were we to remove labor supply considerations altogether by assuming fully rigid nominal wages, the externality would disappear and external

[3] See also Farhi and Werning (2012, 2014, 2017), Korinek and Simsek (2016), Schmitt-Grohe and Uribe (2016), Acharya and Bengui (2018), Fornaro and Romei (2019) and Bianchi and Coulibaly (2021).

[4] For earlier articulations of these ideas in the information economics and general equilibrium literatures, see, e.g., Stiglitz (1982), Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986). In financial economics, see, e.g., Gromb and Vayanos (2002) and Lorenzoni (2008). In international macroeconomics, see Korinek (2007, 2018), Bianchi (2011), Jeanne and Korinek (2010, 2019, 2020), Benigno, Chen, Otrok, Rebucci and Young (2013, 2016), Bengui (2014) and Bianchi and Mendoza (2018). Also, see Coulibaly (2020) and Ottonello (2021) for examples of studies combining pecuniary externalities mattering due to financial frictions with aggregate demand externalities arising from nominal rigidities.

borrowing would become constrained efficient.[5]

The contrast between the macroeconomic externality we focus on and the aggregate demand externalities studied by Farhi and Werning (2016) and others go beyond simple semantics. When aggregate demand externalities lead to inefficient financial decisions in contexts where constraints on price adjustment and monetary policy prevent goods-specific labor wages to be closed, the general prescription for taxes on financial transactions is to incentivize agents to shift wealth toward states of nature where their spending is relatively high on goods whose provision is most depressed. Boosting spending on these goods is something monetary policy would like to achieve, but is unable to do due to constraints such as a fixed exchange rate (Farhi and Werning 2012, 2017, Schmitt-Grohe and Uribe 2016) or a zero lower bound (Farhi and Werning 2016, Korinek and Simsek 2016). This general principle does not apply in our context, where it is usually optimal to tilt spending *away* from the country whose output gap is the most negative. Indeed, in our model, inefficiencies of financial decisions arise even when marginal propensities to consume (MPCs) are identical across agents and over time. Instead of being designed to redirect spending toward relatively more depressed goods, in our setup financial market interventions are motivated by a desire to shift spending away from where it worsens the most unfavorable output-inflation trade-offs through supply-side channels. Our paper hence complements the aggregate demand externality literature by providing an insight specific to circumstances where, as at the current juncture, central banks may not be able to limit inflation without causing economic slowdowns.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 characterizes optimal monetary and capital flow management policy, establishing the constrained inefficiency of the free capital mobility regime. Section 4 makes our results more tangible by studying the world economy's adjustment to a shock generating an output-inflation trade-off. Section 5 discusses possible extensions of our analysis and Section 6 concludes.

# 2 Model

The world is composed of two countries of equal size, Home and Foreign. In each country, households consume goods and supply labor, while firms hire labor to produce output.

[5]We thank Anton Korinek and Ivan Werning for enlightening discussions on the distinction between our externality and both the aggregate demand externalities and pecuniary externalities emphasized in the literature.

Variables pertaining to Foreign are denoted with asterisks.

## 2.1 Households

Each country is populated by a representative household. The preferences of Home's representative household are represented by the utility functional:

$$\int_0^\infty e^{-\rho t}\left[\log C_t - \frac{N_t^{1+\phi}}{1+\phi}\right] dt,$$

where $C_t$ is consumption, $N_t$ is labor supply, $\phi$ is the inverse Frisch elasticity of labor supply, and $\rho$ is the discount rate.[6] The consumption index $C_t$ is defined as

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

$C_{H,t}$ and $C_{F,t}$ are CES aggregates over a continuum of goods produced respectively in Home and Foreign, with elasticity of substitution between varieties produced within a country equal to $\varepsilon > 1$. The elasticity of substitution between domestic and foreign goods is $\eta > 0$ and $\alpha \in (0, 1/2]$ is a home bias parameter capturing the degree of trade openness. When $\alpha = 1/2$, there is no home bias. In contrast, when $\alpha < 1/2$, households' preferences are biased toward domestically produced goods.

Households can trade two types of nominal bonds: an international bond traded internationally and a domestic bond traded only domestically. Domestic bonds are denominated in domestic currency, while the international bond is (arbitrarily) denominated in Home's currency (without loss of generality given perfect foresight).

The Home household's budget constraint is given by:

$$\dot{D}_t + \dot{B}_t = i_t D_t + i_{B,t} B_t + W_t N_t + \Pi_t - \int_0^1 P_{H,t}(l)\, C_{H,t}(l) dl - \int_0^1 P_{F,t}(l) C_{F,t}(l) dl$$

where $D_t$ is domestic bond holdings, $B_t$ is international bond holdings, $i_t$ denotes the return on Home bonds, $i_{B,t}$ denotes the return on the international bond for the Home household, $W_t$ is the nominal wage, $C_{H,t}(l)$ is Home's consumption of good $l$ produced domestically, $C_{F,t}(l)$ is Home's consumption of imported good $l$, $P_{H,t}(l)$ is the price of the good $l$ produced domestically and $P_{H,t}(l)$ is the price of imported good $l$.

[6] Our exposition focuses on Home's representative household, but the environment faced by Foreign's representative household is symmetric.

Households choose consumption, labor supply and bond holdings to maximize utility. Their optimality conditions for labor supply, domestic bond holdings and international bond holdings are given in logarithmic form by

$$w_t - p_t = \phi n_t + c_t, \tag{1}$$

$$\dot{c}_t = i_t - \pi_t - \rho, \tag{2}$$

$$\dot{c}_t = i_{B,t} - \pi_t - \rho, \tag{3}$$

where lower case letters denote natural logarithms of the respective capital letter variables, Home's consumer price index (CPI) follows from standard expenditure minimization,

$$P_t \equiv \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{1/(1-\eta)},$$

where $P_{H,t}$ is Home's PPI, $P_{F,t}$ is Home's price index of imported goods, and $\pi_t \equiv \dot{P}_t / P_t$ is the Home CPI inflation rate. Condition (1) is the Home household's optimality conditions for labor supply, which equate the marginal disutility of work to the real wage. (2) is the Home household's Euler equation for domestic bonds, while (3) is the analogous Euler equation for the international bond.

The Foreign household faces an environment symmetric to that of the Home household. Variables pertaining to the Foreign household are indexed by asterisks. To accommodate possible deviations from perfect capital mobility, we allow for (tax-induced) return differentials across countries on the international bond. In particular, we assume that the return the on international bond has two components: a component that is common across countries $\underline{i}_t$ and a country-specific component ($\tau_t$ for Home and $\tau_t^*$ for Foreign) that captures taxes on international financial transactions financed by lump-sum taxes raised locally. Let $\tau_t^D$ be related to the wedge between the return on the international bond faced by Home and Foreign households by:

$$\tau_t^D \equiv \frac{i_{B,t} - i_{B,t}^*}{2} = \frac{\tau_t - \tau_t^*}{2}. \tag{4}$$

Under free capital mobility, we will have $\tau_t^D = 0$ for all $t \geq 0$. But we will also consider situations where $\tau_t^D \neq 0$ as a result of different taxes across countries ($\tau_t \neq \tau_t^*$). Finally, we assume that countries have symmetric net foreign asset positions (i.e., equal to 0) at time 0.

The optimality conditions of the Foreign household are symmetric to (1)-(3) and given by

$$w_t^* - p_t^* = \phi n_t^* + c_t^*, \tag{5}$$
$$\dot{c}_t^* = i_t^* - \pi_t^* - \rho, \tag{6}$$
$$\dot{c}_t^* = i_{B,t}^* - \dot{e}_t - \pi_t^* - \rho, \tag{7}$$

where $e_t$ is the log of the nominal exchange rate $E_t$, defined as the Home currency price of the Foreign currency, and $\pi_t^* \equiv \dot{P}_t^*/P_t^*$ is the Foreign CPI index. The real exchange rate is defined as the ratio of the two countries' CPI expressed in a common currency, $Q_t \equiv E_t P_t^*/P_t$. Further, letting $P_{F,t}^*$ denote Foreign's PPI and $P_{H,t}^*$ Foreign's price index of imported good, the Home's terms of trade can be defined as $S_t = P_{F,t}/(E_t P_{H,t}^*)$..

Combining (2)-(7) with (4) leads to the distorted interest parity condition

$$i_t = i_t^* + \dot{e}_t + 2\tau_t^D.$$

Under free capital mobility (i.e., when $\tau_t^D = 0$), standard interest parity holds. In contrast, when $\tau_t^D > 0$ the Home household faces a higher borrowing cost, while when $\tau_t^D < 0$ it is the Foreign household who faces a higher borrowing cost.

## 2.2 Firms

**Technology.** Firms in Home and Foreign produce differentiated goods $l \in [0,1]$ with a linear technology: $Y_t(l) = AN_t(l)$, resp. $Y_t^*(l) = A^* N_t^*(l)$, where $A$ and $A^*$ denote productivity parameters normalized to one for convenience. We let $N_t(l)$ (resp. $N_t^*(l)$) be a composite of individual household labor in Home (resp. Foreign) using a constant elasticity of substitution aggregator, where the elasticity of substitution among varieties of domestic labor in each country, $\varepsilon_t^w$ (resp. $\varepsilon_t^{w*}$), is stochastic and common to all firms within the country. The variation in wage markups, $\mu_t^w \equiv \frac{\varepsilon_t^w}{\varepsilon_t^w - 1}$ and $\mu_t^{w*} \equiv \frac{\varepsilon_t^{w*}}{\varepsilon_t^{w*} - 1}$, is the source of cost-push shocks that gives rise to well-known trade-offs between stabilizing economic activity and inflation (see e.g., Clarida, Gali and Gertler 2002, Engel 2011 and Groll and Monacelli 2020).

**Price setting.** Firms operate under monopolistic competition and engage in infrequent price setting à la Calvo (1983). Prices are set in producers' currency and the law of one price holds for each good. Each firm has an opportunity to reset its price when it receives a price-change signal, which itself follows a Poisson process with intensity $\rho_\delta \geq 0$. As a result, a fraction $\delta$ of firms receives a price-change signal per unit of time. These firms reset their price, $P^r_{H,t}(j)$, to maximize the expected discounted profits

$$\int_t^\infty \rho_\delta e^{-\rho_\delta(k-t)} \frac{\lambda_k}{\lambda_t} \left[P^r_{H,t}(j) - P_{H,k} MC_k\right] Y_{k|t} dk,$$

subject to the demand for their own good, $Y_{k|t} = \left(P^r_{H,t}/P_{H,k}\right)^{-\varepsilon} Y_k$, taking as given the paths of domestic output $Y$, of the domestic PPI $P_H$, and of the domestic real marginal cost $MC$. The real marginal cost is defined as $MC_k \equiv (1-\tau^N) W_k/(A_k P_{H,k})$, where $\tau^N$ is a time-invariant labor subsidy.[7] $\lambda_k$ denotes the Home household's time $k$ marginal utility of consumption, so that the ratio $\lambda_k/\lambda_t$ is the firm's relevant discount factor between time $t$ and time $k \geq t$. The pricing environment is symmetric in Foreign. In the limiting case of flexible prices (i.e. $\rho_\delta \to \infty$), firms are able to reset their prices continuously and optimal pricing setting reduces to $P_{H,t} = (1-\tau^N)\frac{\varepsilon}{\varepsilon-1} W_t$.

## 2.3 Equilibrium Dynamics

Given paths for interest rates and taxes on international financial transactions, an equilibrium is a constellation where all households and firms optimize and markets clear.

**International consumption smoothing.** Combining the Home and Foreign households' Euler equations for the international bonds yields an international consumption smoothing condition relating the ratio of marginal utility in both countries to the real exchange rate

$$C_t = \Theta_t Q_t C_t^*, \tag{8}$$

where $\Theta_t \equiv \Theta_0 \exp\left[\int_0^t \xi_s ds\right]$ captures relative spending by Home consumers, with $\Theta_0$ being a constant related to initial relative wealth positions.[8] Condition (8) indicates that under free capital mobility (i.e., absent taxes on international financial transactions), Home and Foreign spending are related to each other through a time-invariant coefficient of proportionality $\Theta_0$. Taxes on international financial transactions alter capital flows between the two countries and make relative spending $\Theta_t$, sometimes referred to as *demand imbalance* in the literature, a time-varying object. Taking logs on both sides of (8), and taking into

[7] As is standard in the New Keynesian literature, we assume that this subsidy is set at the level that would be optimal in a steady state with flexible prices. This subsidy can thus be thought of as offsetting long-run distortions stemming from monopolistic competition.

[8] In models featuring uncertainty and complete markets, this condition is often labeled as an international risk sharing condition.

account the (first-order accurate) relationship between the real exchange rate and the terms of trade, $q_t = (1-2\alpha)s_t$, we obtain the log-linearized international consumption smoothing condition

$$c_t - c_t^* = \theta_t + (1-2\alpha)s_t. \tag{9}$$

**Output determination.** Market clearing for a good $l$ produced in Home requires that the supply of the good equals the sum of the demand emanating from Home and Foreign:

$$Y_t(l) = \underbrace{(1-\alpha)\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t}_{C_{H,t}(l)\text{: Home demand for Home variety } l} + \underbrace{\alpha\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*}_{C_{H,t}^*(l)\text{: Foreign demand for Home variety } l}. \tag{10}$$

At the level of Home's aggregate output, market clearing hence requires

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)\,C_t + \alpha Q_t^{\eta} C_t^*\right].$$

A first order approximation of this condition around the symmetric steady state yields the following log-linear expression:

$$y_t = (1-\alpha)\left[c_t + \alpha\eta s_t\right] + \alpha\left[c_t^* + (1-\alpha)\eta s_t\right]. \tag{11a}$$

Similarly, the log-linearized Foreign goods market clearing condition is given by

$$y_t^* = (1-\alpha)\left[c_t^* - \alpha\eta s_t\right] + \alpha\left[c_t - (1-\alpha)\eta s_t\right]. \tag{11b}$$

These expressions indicate that output in each country depends on consumption in Home and Foreign, as well as on the terms of trade: a terms of trade improvement for Home (i.e., a decrease in $s_t$) raises demand for output in Foreign at the expense of demand for output in Home via the expenditure switching channel.

Combining the consumption smoothing relation (9) with the market clearing conditions (11a) and (11b) yields an expression satisfied by the equilibrium terms of trade given by

$$y_t - y_t^* = \omega s_t + (1-2\alpha)\theta_t, \tag{12}$$

where $\omega \equiv \eta - (\eta-1)(1-2\alpha)^2 > 0$. The expression indicates that output is relatively higher in the country which has less favorable terms of trade or, in the presence of home bias in consumption, in the country benefiting from a positive demand imbalance. In the absence of home bias (i.e., when $\alpha = 1/2$), since the composition of consumption is identical across the two countries, demand imbalances do not translate into differences in output. Combining the budget constraints of households, firms, as well as the condition relating the equilibrium terms of trade to relative output (12), we arrive at an expression for the trade balance,

$$nx_t = \frac{\omega - 1}{2} s_t - \alpha\theta_t, \tag{13}$$

which show that the effects of an appreciated terms of trade on the trade balance depends on the relative importance of the elasticity of substitution across goods ($\eta$) and the intertemporal elasticity of substitution, which in our specification is equal to one. Furthermore, all else equal, a positive demand imbalance ($\theta_t > 0$) brings about capital inflows (i.e., reduces the trade balance).

Denoting aggregate output in Home as $Y_t \equiv \left[\int_0^1 Y_t(l)^{(\varepsilon-1)/\varepsilon} dl\right]^{\varepsilon/(\varepsilon-1)}$, aggregate Home employment relates to aggregate Home output according to $N_t \equiv \int_0^1 N_t(l)dl = Y_t Z_t$, where $Z_t \equiv \int_0^1 (P_t(l)/P_t)^{-\varepsilon} dl$. Since equilibrium variations in $z_t \equiv \ln Z_t$ around the steady state are of second order, up to a first order, the relationships between aggregate employment and output in the two countries are given by:

$$n_t = y_t, \quad \text{and} \quad n_t^* = y_t^*. \tag{14}$$

**Inflation and marginal costs.** Under our Calvo price setting assumption, up to a first-order approximation, the dynamics of PPI inflation in each region are described by

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \kappa \widehat{mc}_t, \tag{15a}$$

$$\dot{\pi}_{F,t}^* = \rho \pi_{F,t}^* - \kappa \widehat{mc}_t^*. \tag{15b}$$

where $\kappa \equiv \rho_\delta(\rho + \rho_\delta)$, and $\widehat{mc}_t$ (resp. $\widehat{mc}_t^*$) denotes the log deviation of the real marginal cost from its steady state value. Using the aggregate production functions (14) and the labor supply equations (1), these are given by

$$\widehat{mc}_t = (1+\phi) y_t - \frac{\omega - 1}{2} s_t + \alpha\theta_t + u_t, \tag{16a}$$

$$\widehat{mc}_t^* = (1+\phi) y_t^* + \frac{\omega - 1}{2} s_t - \alpha\theta_t + u_t^*. \tag{16b}$$

Intuitively, the real marginal cost (measured in units of the domestic good) depends positively on the marginal rate of substitution between consumption and leisure and negatively on the terms of trade.[9] However, since the equilibrium marginal rate of substitution itself depends ambiguously on the terms of trade (for given levels of output and the demand imbalance), the relationship between the terms of trade and the marginal cost is a priori ambiguous. Finally, for given levels of output and the terms of trade, a positive demand imbalance raises the marginal rate of substitution of a country's residents' and thus increases the domestic firms' marginal cost.[10] The cost-push shocks, $u_t \equiv \mu_t^w - \mu^w$ and $u_t^* \equiv \mu_t^{w*} - \mu^w$, are deviations of wage markups from their steady state value.

## 2.4 World and Difference formulation

Before studying optimal policy, it is convenient to rewrite the dynamics of output and inflation in both regions in "world" and "difference" format. We respectively define the world output and the cross-country output differential as $y_t^W = (y_t + y_t^*)/2$ and $y_t^D = (y_t - y_t^*)/2$. Similarly, we define the world PPI inflation and cross-country PPI inflation differential as $\pi_t^W = (\pi_{H,t} + \pi_{F,t}^*)/2$ and $\pi_t^D = (\pi_{H,t} - \pi_{F,t}^*)/2$. Combining the expressions for PPI inflation dynamics (15a)-(15b) with the marginal cost expressions (16a)-(16b) yields New-Keynesian Phillips curves (NKPC) for world and differences

$$\dot{\pi}_t^W = \rho \pi_t^W - \kappa(1+\phi) y_t^W - \kappa u_t^W, \tag{17}$$

$$\dot{\pi}_t^D = \rho \pi_t^D - \kappa \left[ (1+\phi) y_t^D - \frac{\omega - 1}{2} s_t + \alpha \theta_t \right] - \kappa u_t^D. \tag{18}$$

The equilibrium terms of trade expression (12) can also be re-written as

$$2 y_t^D = \omega s_t + (1 - 2\alpha)\theta_t. \tag{19}$$

## 2.5 Gaps from efficient allocation

Since the only shocks we consider are cost-push shocks driven by inefficient mark-up fluctuations, the efficient allocation is time-invariant. Given our productivity normalization, output, employment, consumption and the terms of trade are all equal to one in this

[9] That is to say, an improvement in a country's terms of trade lowers its producers' marginal cost. A terms of trade improvement raises the price of the domestic good relative to that of the consumption basket. Noting that $p_t = p_{H,t} + \alpha s_t$, the labor supply equation (1) implies that the real wage expressed in terms of the domestic good must be equal to $w_t - p_{H,t} = \phi n_t + \sigma c_t + \alpha s_t$, so that the real marginal cost is given by $\widehat{mc}_t = \phi n_t + \sigma c_t - a_t + \alpha s_t$.

[10] Note that for Home, improved terms of trade correspond to a lower $s_t$ while a positive demand imbalance corresponds to a higher $\theta_t$. In contrast, for Foreign, improved terms of trade correspond to a higher $s_t$ while a positive demand imbalance corresponds to a lower $\theta_t$.

efficient allocation (see Appendix A). Therefore, log deviations of variables from their steady state values can also be interpreted as gaps from the efficient allocation.

# 3 Characterization of optimal policy

We now characterize optimal monetary and capital flow management policy in the model. We will see that when monetary policy faces an output-inflation trade-off, a free capital mobility regime is generically constrained inefficient due a macroeconomic externality operating via firms' marginal costs. Under a condition weaker than the well-known Marshall-Lerner condition, capital flows toward the region experiencing the deepest monetary policy-induced recession are excessive. In addition, if, as is empirically plausible, the intra-temporal elasticity of substitution between Home and Foreign goods is larger than one, capital flows in the wrong direction.

## 3.1 Welfare-based loss function

To capture the various trade-offs to be resolved by optimal policies, we use a standard welfare-based loss function. To obtain this loss function, we take a second-order approximation of a symmetrically weighted average of households' utilities in Home and Foreign (see Appendix A).[11] The instantaneous loss function is given by

$$\begin{aligned} L_t = & \left[(1+\phi)\left(y_t^W\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_t^W\right)^2\right] + \left[(1+\phi)\left(y_t^D\right)^2 + \frac{\varepsilon}{\kappa}\left(\pi_t^D\right)^2\right] \\ & + \alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + \alpha(1-\alpha)\left[\theta_t - (\eta-1)(1-2\alpha)s_t\right]^2, \end{aligned} \tag{20}$$

where the output gap and inflation are again expressed in "world" and "difference" forms. The first two terms in (20) featuring squared output gaps and inflation reflect sticky price distortions familiar from the closed economy literature. The third and fourth terms, reflecting distortions specific to the open economy context, capture welfare losses stemming from an inefficient cross-country distribution of consumption potentially caused by two factors: the demand imbalance $\theta_t$ and the terms of trade gap $s_t$.[12]

Normative research in New Open-Economy Macroeconomics (e.g., Benigno 2009) has traditionally studied how monetary policy should be set in a context where the demand

[11] Given equal country sizes, our adoption of equal welfare weight can be interpreted as an implicit assumption of perfect insurance with respect to the risk of the cost-push shock scenario described above.

[12] This later factor, however, disappears in a widely studied special case featuring unit elasticities.

imbalance term $\theta_t$ was equal to zero (i.e., under complete markets) or was endogenously responding to shocks and other macroeconomic variables (i.e., under incomplete markets). Our approach, in contrast, is to treat the demand imbalance $\theta_t$ as a policy variable and ask whether actively managing it may be desirable in a context where it could otherwise be left at zero.

## 3.2 Optimal monetary policy

The optimal monetary policy problem consists in choosing a path for the welfare relevant output gaps $y_t^W, y_t^D$, inflation $\pi_t^W, \pi_t^D$, and terms of trade $s_t$, to minimize the present value of the loss (20), subject to the NKPCs (17)-(18) and the equilibrium terms of trade expression (19).[13] We have the following characterization.

**Proposition 1** (Optimal monetary policy)**.** *Optimal monetary policy is characterized by the following targeting rules:*

$$\dot{y}_t^W + \varepsilon \pi_t^W = 0 \tag{21}$$

$$\dot{y}_t^D + \varepsilon \pi_t^D = 0 \tag{22}$$

*Proof.* See Appendix B.1. □

This description of optimal cooperative monetary policy is analogous to that commonly encountered for complete markets open-economy models with producer currency pricing (PCP) in the literature. (21) and (22) indicate that, both in "world" and "difference" terms, optimal policy strikes a balance between losses from inflation and losses from deviations of output from its efficient level. The two targeting rules can be combined to deliver targeting rules for each country that only depend on the domestic output gap and PPI inflation, i.e., $\dot{y}_t + \varepsilon \pi_{H,t} = 0$ and $\dot{y}_t^* + \varepsilon \pi_{F,t}^* = 0$, a feature referred to as *inward looking* monetary policy in the New Open-Economy Macroeconomics literature. It is worth stressing that this characterization does not rely on any particular assumption regarding the path of $\theta_t$ (other than it being exogenous, or chosen by policy). In particular, it holds under free capital mobility (i.e., $\theta_t = 0 \ \forall t$), as well as under an optimally managed capital account regime to be derived below.

[13] Implicitly, in line with the literature, we assume that the policymaker has access to a date 0 transfer so the optimal policy problem reflects efficiency rather than a mix of efficiency and redistributive considerations. For a formal statement of the optimal monetary policy problem, see Appendix B.1.

The targeting rules (21) and (22) lead us to one observation, summarized in the corollary below, which helps us narrow down the role played by capital flows in response to shocks.

**Corollary 1** (Irrelevance of capital flow regime for world variables)**.** *The paths of world output gap $y_t^W$ and world inflation $\pi_t^W$ are independent of the capital flow regime (i.e., of the path of $\theta_t$).*

*Proof.* See Appendix B.2 □

This observation follows directly from combining the "world" NKPC (17) with the "world" monetary policy targeting rule (21) and means that the capital flow regime only matters for the determination of cross-country "difference" variables and the terms of trade.[14] Therefore, both from a positive and normative standpoint, an analysis of the role played by capital flows in the adjustment to shocks can legitimately center on the dynamics of cross-country difference variables $y_t^D$, $\pi_t^D$ and external variables $s_t$ and $\theta_t$.

**Remark** (Inward vs. outward looking monetary policy)**.** *When the path of the demand imbalance $\theta_t$ deviates from zero, asset markets are no longer complete and the inward lookingness of monetary policy in* (21)-(22) *contrasts with the* outward looking *rules derived in studies assuming other forms of market incompleteness, such as financial autarky or a single bond in environments featuring uncertainty (e.g., Corsetti, Dedola and Leduc 2010, 2018).*[15] *In these studies, the demand imbalance is an endogenous variable whose fluctuations depend on the interaction of shocks and other variables influenced by monetary policy (such as the cross-country difference in the output gap). As a result, monetary policy can manage distortions caused by market incompleteness and generally chooses to do so, resulting in outward looking rules. In our case, in contrast, the demand imbalance is either exogenous or directly controlled by policy, so there is no scope for monetary policy to manage market incompleteness distortions, hence the inward looking rules.*

## 3.3 Optimal capital flow management

To question the constrained efficiency of the free capital mobility regime, we make the demand imbalance $\theta_t$ a choice variable of the optimizing policy maker and ask under which circumstances $\theta_t$ is set to a value different from zero. The optimal policy problem now consists in choosing a path for the welfare relevant output gaps $y_t^W, y_t^D$, inflation

[14]See Groll and Monacelli (2020) for a similar result regarding the irrelevance of the exchange rate regime for the determination of "world" variables.

[15]The literature refers to outward looking monetary policy when targeting rules in open-economy models also feature external variables, such as international relative prices or the demand imbalance term.

$\pi_t^W, \pi_t^D$, terms of trade $s_t$ *and* demand imbalance $\theta_t$ to minimize the present discounted value of the loss (20), subject to the NKPCs (17), (18) and the equilibrium terms of trade relation (19).[16]

In addition to the targeting rules associated with monetary policy, (21) and (22), optimal policy now also pertains to an additional capital flow management margin.

**Proposition 2** (Optimal capital flow management)**.** *The optimal capital flow regime is characterized by the targeting rule*

$$\theta_t = \frac{\chi - (1 - 2\alpha)}{\chi} 2y_t^D. \tag{23}$$

*where $\chi \equiv 2(1 - \alpha)\eta$ denotes the trade elasticity.*[17]

*Proof.* See Appendix B.3 □

To the extent that shocks generating an output-inflation trade-off generally result in a non-zero cross-country difference in output gaps according to (18)-(19)-(22), the targeting rule (23) indicates that the relative demand imbalance is only always set to zero when the trade elasticity $\chi$ is equal to the degree of home bias $1 - 2\alpha$. Away from this knife-edge case, the optimally managed demand imbalance is generally not zero. Therefore, the free capital mobility regime is generically constrained inefficient when monetary policy faces an output-inflation trade-off, and it is only constrained efficient under a special parametric condition stating that the trade elasticity is identically equal to the degree of home bias. To our knowledge, this condition has not received any attention in the literature.

In favor of which country should policy want to tilt spending? (23) suggests that that the answer to this question depends on the relative size of the trade elasticity $\chi$ and the degree of home bias $1 - 2\alpha$. When $\chi < 1 - 2\alpha$, optimal capital flow management generates a demand imbalance in favor of the country with the smallest output gap, while when $\chi > 1 - 2\alpha$, optimal capital flow management generates a demand imbalance in favor of the country with the largest output gap. In this latter case, which appears to be the

[16]See Appendix B for a formal statement of the problem.

[17]The trade elasticity $\chi$ is defined as the sum of the absolute values of the price elasticity of imports and the price elasticity of exports, holding aggregate consumption constant. Formally,

$$\chi \equiv \left.\frac{-\partial \log C_{F,t}}{\partial \log P_{F,t}/P_{H,t}}\right|_{C_t} + \left.\frac{-\partial \log C_{H,t}^*}{\partial P_{H,t}^*/P_{F,t}^*}\right|_{C_t^*} = 2(1-\alpha)\eta.$$

most relevant one empirically, capital flows toward the country whose activity is the most depressed are excessive.[18]

What is the economics behind this inefficiency of the free capital mobility regime? We next argue that it has to do with a *macroeconomic externality* operating via firms' marginal costs when monetary policy faces an output-inflation trade-off.

### 3.4 Macroeconomic externality via firms' marginal costs

To nail down the inefficiencies at work in the free capital mobility regime, it is useful to ask how a marginal deviation from the equilibrium external borrowing positions in that regime alter the constraints faced by monetary policy and hence aggregate outcomes.

Consider a marginal increase in borrowing by Home from Foreign at instant $t$ (i.e., $\theta_t = \epsilon$ for some small $\epsilon$, leaving $\theta_k = 0$ for all other $k \neq t$).[19] Using (19) to substituting for the equilibrium terms of trade in the marginal cost expressions (16a)-(16b) and applying the envelope theorem, the change in the loss function induced by this perturbation is given by

$$\frac{dL_t}{d\theta_t} = \varphi_t^D \frac{\partial mc^D(y_t^D, \theta_t)}{\partial \theta_t}, \tag{24}$$

where $\varphi_t^D$ is the co-state variable associated with the NKPC in differences (18). (24) shows that the marginal increase in borrowing by Home from Foreign affects global welfare losses via its effects on the cross-country "difference" in marginal costs. Now, observe that the cross-country difference in marginal costs can be decomposed into two components

$$mc^D(y_t^D, \theta_t) = \underbrace{\left[\phi + \frac{1-2\alpha}{\omega}\right] y_t^D + \frac{\alpha\chi}{\omega}\theta_t}_{\text{difference in real wages}} + \underbrace{\frac{\alpha}{\omega}\left[2y_t^D - (1-2\alpha)\theta_t\right]}_{\text{differences in purchasing power}} + u_t^D. \tag{25}$$

The first component reflects cross-country differences in labor costs arising from differences in the real wage (in terms of each country's consumption bundle). The second component reflects cross-country differences in purchasing power arising from movements in the

[18]Most calibrations of the the model place the trade elasticity above one, in which case the condition $\chi > (1-2\alpha)$ is necessarily satisfied.

[19]For the sake of the argument, we assume that this increase in borrowing is compensated by a change in the date 0 implicit transfer. More generally, what matters for the externality to matter is that the balancing transaction occurs at a time when the government's multiplier on the NKPC (18) has a value different from the one at time $t$.

terms of trade. The marginal cost derivative in (24) is therefore given by

$$\frac{\partial mc^D(y_t^D,\theta_t)}{\partial\theta_t} = \frac{\alpha\chi}{\omega}\left[\underbrace{1}_{\text{real wage effect}} - \underbrace{\frac{(1-2\alpha)}{\chi}}_{\text{purchasing power effect}}\right], \tag{26}$$

where the two terms reflect the two aforementioned components of the cross-country difference in marginal costs. First, raising Home consumption and lowering Foreign consumption shifts up labor supply in Home while shifting it down in Foreign. In equilibrium, this leads to a rise in Home's real wage and a drop in Foreign's real wage, thereby raising the cross-country difference in marginal costs. Second, in the presence of home bias in preferences ($\alpha < 1/2$), the appreciation of the terms of trade induced by the increase in borrowing by Home from Foreign raises Home's purchasing power while decreasing Foreign's purchasing power. This lowers marginal costs in Home and raises them in Foreign, hence reducing the cross-country difference. The strength of this effect is proportional to the ratio of the degree of home bias $1-2\alpha$ to the trade elasticity $\chi$. On the one hand, the stronger the home bias, the more changes in relative spending affect the relative price between Home and Foreign good.[20] On the other hand, the larger the trade elasticity, the smaller are price movements associated to a given change in relative spending.

When home bias is low or when the trade elasticity is high, $\chi > (1-2\alpha)$, the real wage effect dominates and an increase in $\theta_t$ raises the cross-country difference in marginal costs. In contrast, when home bias is high or the trade elasticity is low, $\chi < (1-2\alpha)$, the purchasing power effect dominates and the increase in $\theta_t$ lowers the cross-country difference in marginal costs. The two effects exactly cancel out in the knife-edge case where $\chi = (1-2\alpha)$, in which case the difference in marginal costs independent of $\theta_t$.

These effects of marginal changes in external borrowing work in general equilibrium as prices adjust in goods and labor markets. As a result, they are ignored by atomistic agents. Yet, when the output-inflation trade-off is more stringent in one of the two countries, i.e. when $\varphi_t^D \neq 0$, a marginal increase in borrowing by Home from Foreign at instant $t$ generates a first-order welfare effect by tightening or relaxing the constraint faced by the monetary authority, as indicated by (24).

We next argue that the externality just discussed appears to be powerful enough that it may well result in trade imbalances of opposite signs in the constrained efficient regime

[20] Absent home bias ($\alpha = 1/2$), since households in both countries consume the exact same basket, changes in relative spending are not associated to changes in relative prices.

and in the free capital mobility regime.

### 3.5 Topsy-turvy capital flows

Combining the targeting rule (23) with the equilibrium terms of trade expression (19), we obtain that the terms of trade is proportional to the cross-country difference in the output gap in the constrained efficient regime, $s_t = 2y_t^D/\chi$, albeit with a different coefficient than under free capital mobility, where the relationship follows from (19) (with $\theta_t = 0$) and is given by $s_t = 2y_t^D/\omega$. Substituting these terms of trade expressions into the net export expression (13), we obtain a trade balance of

$$nx_t = -\frac{2\alpha}{\chi}\, y_t^D \tag{27}$$

under the constrained efficient regime, while the trade balance under free capital mobility is given by:

$$nx_t = \frac{\omega - 1}{\omega} y_t^D. \tag{28}$$

This points to qualitatively different patters of trade imbalances under the two regimes, which we summarize in the following proposition.

**Proposition 3** (Topsy-turvy capital flows)**.** *In the constrained efficient capital account regime, the country with the most depressed output always runs a trade surplus. This contrasts with the free capital mobility regime, where the country with the most depressed output runs a trade deficit if $\eta > 1$ and a trade surplus if $\eta < 1$.*

*Proof.* The proof follows directly from (27), (28), and the definitions of $\omega$ and $\chi$. □

The proposition implies that in the presence of cross-country differences in the severity of (policy induced) recessions, capital flows are *topsy-turvy* under free capital mobility in the empirically plausible case where $\eta > 1$. That is, the country with the most depressed activity runs a trade deficit under free capital mobility while it really should be running a trade surplus. Hence, rather than simply causing capital flows to be excessive, the macroeconomic externality discussed in Section 3.4 is likely to be strong enough to flip their direction.

**Neoclassical and Keynesian motives of inter-temporal trade.** To understand the essence of Proposition 3, it is useful to decipher the various motives for inter-temporal trade in the model. In the free capital mobility regime, these motives are purely neoclassical and are well understood since at least Cole and Obstfeld (1991): A temporarily lower income in Home creates an incentive to borrow, but the terms of trade appreciation accompanying this lower income generates an incentive to save. When the intra-temporal elasticity is high (i.e. $\eta > 1$), terms of trade movements are muted, and the first effect dominates. When the intra-temporal elasticity is low (i.e. $\eta < 1$), terms of trade movements are strong, and the second effect dominates. And when the intra- and inter-temporal elasticities are equal, the two effects neutralize each other.

In the constrained efficient regime, an additional *Keynesian macroeconomic stabilization motive* is also present. This motive calls for relaxing the output-inflation trade-off in the country where it is the least favorable. For the sake of illustrating the scope for topsy-turvy capital flows, consider the case of the Cole-Obstfeld parameter specification ($\eta = 1$). As we just argued, in this case the two neoclassical motives cancel out and result is zero trade imbalances under free capital mobility. In the constrained efficient capital flow regime, the Keynesian motive in addition generates an incentive to save for the country with the lowest output, as $\chi > (1 - 2\alpha)$ holds (see Proposition 2 and discussion of Section 3.4). That country thus experiences a trade surplus. For $\eta$ slightly above one, the net neoclassical effect becomes positive, generating a trade deficit by the country with the most negative output gap under free capitail mobility, but in the constrained efficient regime the Keynesian effect still dominates to yield a trade surplus by that country. As $\eta$ is raised further, the net neoclassical effect grows stronger in both regime, but it happens to never overturns the Keynesian effect in the constrained efficient regime.[21]

This topsy-turvy capital flows result is represented graphically in Figure 2. At a given point in time, we can define the most afflicted country to be the country whose output is the most depressed.[22] For any pair of values for $\alpha$ and $\eta$, the figure shows whether capital flows are excessive, insufficient and topsy-turvy. The red Cole-Obstfeld line depicts cases where capital flows are zero under free capital mobility. Above this line, under free capital mobility, capital flows from the least afflicted country, which runs a trade surplus, to the most afflicted country, which runs a trade deficit. These flows are excessive and go in the wrong direction when compared to those that would prevail in the constrained

[21]In the limit where $\eta \to \infty$, as long as $\alpha > 0$, the trade balance becomes proportional to the difference in the output gap under free capital mobility, $nx_t = y_t^D$, but converges to zero in the constrained efficient regime, $nx_t = 0$.

[22]Since each country's output gap is proportional to the policymaker's co-state on the NKPCs, the most afflicted country also happens to be the one for which relaxing the NKPC is the most valuable to the policymaker.

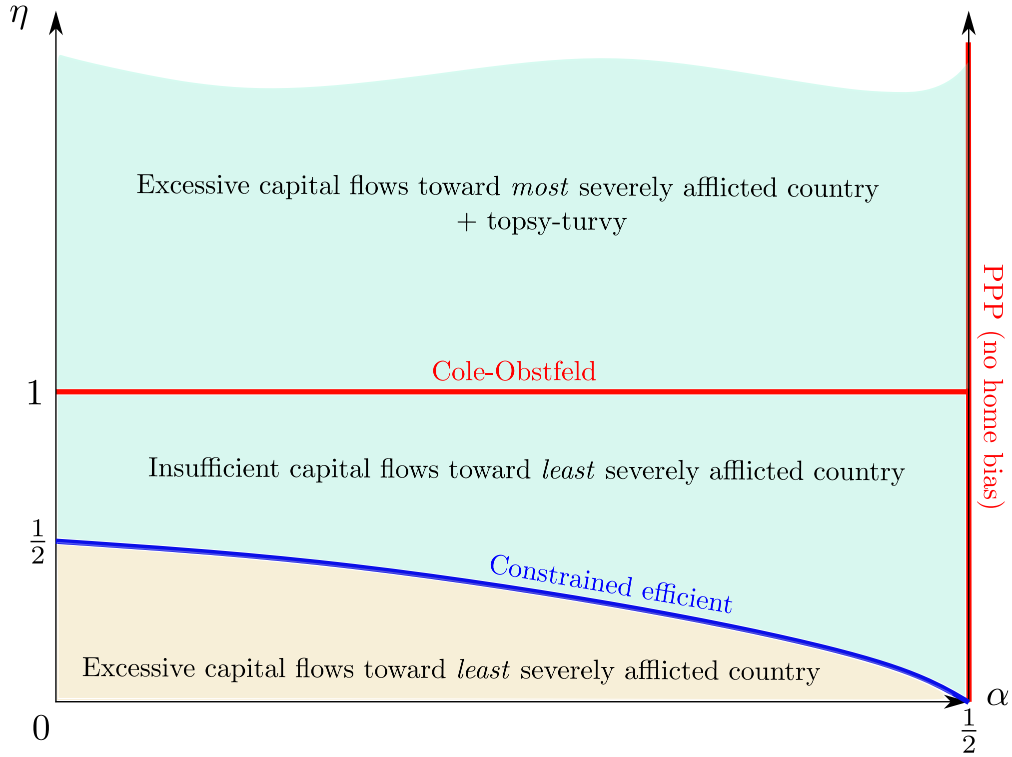


Figure 2: Characterization of distortions to capital flows in free capital mobility regime.

efficient regime. Below the Cole-Obstfeld line, capital flows in the opposite direction. Within this area, the blue concave curve depicts the knife-edge cases where the free capital mobility regime is constrained efficient. In the area above the concave curve (but below the Cole-Obstfeld line), capital flows from the most to the least afflicted country, but to an *insufficient* extent. In contrast, in the area under the concave curve, capital flows from the most to the least afflicted country in an *excessive* way. As alluded to earlier, the model calibrations typically used in the literature are situated in the area above the Cole-Obstfeld curve (see Section 4.3 for an illustration).

### 3.6 Insights from two special cases

Two leading special cases of the model, both extensively studied in the literature, reveal interesting insights.

**No home bias and MPC homogeneity.** In the first one, which the early New Open-Economy Macroeconomics literature almost exclusively focused on (see, e.g., Clarida et al. 2002 or Benigno and Benigno 2003), home bias is abstracted from ($\alpha = 1/2$) and, as a result, purchasing power parity (PPP) holds (i.e., $q_t = 0$). The free capital mobility outcome features an international equalization of consumption at all times, despite possible divergences in economic activity (i.e., (9) with $\theta_t = 0$ implies $c_t = c_t^*$). Yet, this regime is never constrained efficient: according to the targeting rule (23), relative spending should be distorted away from the region with the most depressed output, $\theta_t = 2y_t^D$.

Since this case features identical MPCs on each good across countries (and over time), it clarifies that the intervention motive present in our context with an output-inflation trade-off is conceptually distinct from the one emphasized by Farhi and Werning (2016) and others based on the idea of tilting spending in favor of agents with relatively higher MPCs on more depressed goods. In our context, capital flows alter the conditions faced by monetary policy through the economy's supply side rather than through the demand side. Since absent home bias, the purchasing power effect in (26) is absent, capital flows only affect marginal costs via the real wage through a wealth effect on labor supply. Had we assumed fully rigid wages, households' labor supply behavior would not have any implication for firms' marginal costs and there would be no externality nor inefficiency.

**Cole-Obstfeld, net flows and spillovers.** The second special case, commonly referred to as Cole and Obstfeld (1991) parametrization and popularized in the New Open Economy Macroeconomics literature by Corsetti and Pesenti (2001), features unitary elasticities ($\eta = 1$). This case is remarkable for at least two reasons: under free capital mobility, it features neither external imbalances nor any spillovers from cost-push shocks or monetary policy. Indeed, in this regime net exports in (28) are always zero and both output and inflation only respond to domestic shocks. Yet, we find that this regime is never constrained efficient either: according to the targeting rule (23), relative spending should again be distorted away from the region with the most depressed output, $\theta_t = y_t^D/(1-\alpha)$.

This special case hence illustrates that the macroeconomic externality we document can generate inefficient intertemporal trade even in the absence of any external imbalances and, thus, that external imbalances can be larger under the constrained efficient regime than under free capital mobility. This clarifies that capital mobility is not harmful *per se*. Instead, it is the extent of intertemporal trade that is inefficient because market prices not accurately reflecting the social value of external borrowing in the presence of an output-inflation trade-off.

Furthermore, since the free capital mobility regime is constrained inefficient even in the absence of any spillovers from cost-push shocks or monetary policy, the inefficiency is clearly not related to the correction or internalization of such spillovers.

### 3.7 Decentralization with taxes on capital flows

Several financial policies could be used by a global policymaker to implement the constrained efficient capital flow regime. One possible implementation is through taxes on capital flows. An explicit expression linking these taxes to inflation can be obtained from the targeting rules characterizing optimal monetary and capital flow management policy. From the consumption risk-sharing condition (8), the tax differential satisfies $\tau_t^D = \dot{\theta}_t/2$. Using the targeting rules (22)-(23), it can therefore be related to the cross-country difference in inflation as

$$\tau_t^D = -\frac{\varepsilon}{\chi}\left[\chi - (1-2\alpha)\right]\pi_t^D. \tag{29}$$

Hence, when $\chi > 1 - 2\alpha$, the tax is higher in the country with the lowest PPI inflation rate, while when $\chi < 1 - 2\alpha$, the opposite is true. In the knife-edge case where $\chi = 1 - 2\alpha$ and the free capital mobility regime is constrained efficient, the tax is obviously zero.

## 4 External adjustment to cost-push shocks

To make the uncovered inefficiencies more tangible, let us consider the world economy's adjustment to an unanticipated temporary cost-push (supply) shock that gives rise to an output-inflation trade-off of unequal stringency in the two countries.

**Cost-push shock scenario.** For concreteness, suppose that Home is subject to an inflationary cost-push shock such that $u_t = 2\bar{u} > 0$ for some $\bar{u} > 0$ for $t \in [0, T)$ and $u_t = 0$ for $t \geq T$, while Foreign is not hit by any shock (i.e., $u_t^* = 0$ for $t \geq 0$). In terms of the "world" and "difference" shocks appearing in (17) and (18), we therefore have

$$u_t^W = u_t^D = \begin{cases} \bar{u} > 0 & \text{for} \quad t \in [0, T) \\ 0 & \text{for} \quad t \geq T. \end{cases} \tag{30}$$

As is well understood, monetary policy will not able to perfectly stabilize all variables under this scenario. Instead, it will trade off output gap and inflation distortions, as emphasized in Section 3.2. The main advantage of the step-function scenario in (30) is to allow a sharp graphical characterization of this adjustment under our two capital account regimes of interest.

## 4.1 Adjustment under free capital mobility

In the free capital mobility regime, $\theta_t = 0\ \forall t \geq 0$. Accounting for this fact when substituting the equilibrium terms of trade expression (19) into the NKPC in difference (18) yields a dynamic equation for the cross-country difference in inflation as a function of itself and the cross-country difference in the output gap:

$$\dot{\pi}_t^D = \rho \pi_t^D - \kappa \left( \frac{1}{\omega} + \phi \right) y_t^D - \kappa u_t^D. \tag{31}$$

Meanwhile, differentiating the targeting rule (22) with respect to time yields a dynamic equation for the cross-country difference in the output gap as a function of the cross-country difference in inflation:

$$\dot{y}_t^D = -\varepsilon \pi_t^D. \tag{32}$$

(31) and (32) form a dynamical system in $\pi_t^D$ and $y_t^D$ whose solution encapsulates the dynamics of the cross-country block of the model. $\pi_t^D$ is a jump variable, and although $y_t^D$ could in principle jump, under the optimal plan it is predetermined at $y_0^D = 0$.[23] The system is thus saddle-path stable and the solution can be conveniently represented in a phase diagram. The $\dot{y}_t^D = 0$ locus is described by $\pi_t^D = 0$, while the $\dot{\pi}_t^D = 0$ locus is described by $\rho \pi_t^D = \kappa \left( \frac{1}{\omega} + \phi \right) y_t^D + \kappa u_t^D$. Given our shock scenario, in the $(y_t^D, \pi_t^D)$ space, the $\dot{y}_t^D = 0$ locus is therefore always a flat line at 0, while the $\dot{\pi}_t^D = 0$ locus is an upward sloping straight line with slope $\kappa \left( \frac{1}{\omega} + \phi \right) / \rho$ and intercept $\kappa \bar{u} / \rho > 0$ in the short-run (i.e., for $t \in [0, T)$) and intercept 0 in the long-run (i.e., for $t \geq T$).

The loci are represented in Figure 3, where $y_t^D$ rises (diminishes) south (north) of the $\dot{y}_t^D = 0$ locus and $\pi_t^D$ rises (diminishes) west (east) of the $\dot{\pi}_t^D = 0$ locus. The fictional saddle-path associated with the system being permanently governed by the short-term loci is represented by the upper dashed upward sloping line, while that associated with the system being permanently governed by the long-term loci is represented by the lower dashed upward sloping line. The actual saddle path is represented by the thick curve with arrows.

The inflationary cost-push shock in Home naturally drives a cross-country difference in inflation up on impact. But the initial jump in the inflation difference is limited by monetary policy's commitment to generate a more negative output gap in Home than in Foreign in the future, with the difference in the output gap displaying a hump shape. To

[23] The co-state variable $\varphi_t^D$ is backward looking with an initial condition $\varphi_0^D = 0$, and both $y_t^D$ and $s_t$ are proportional to $\varphi_t^D$ (see equations (A.20), (A.24) and (A.25) with $\theta_t = 0\ \forall t$).

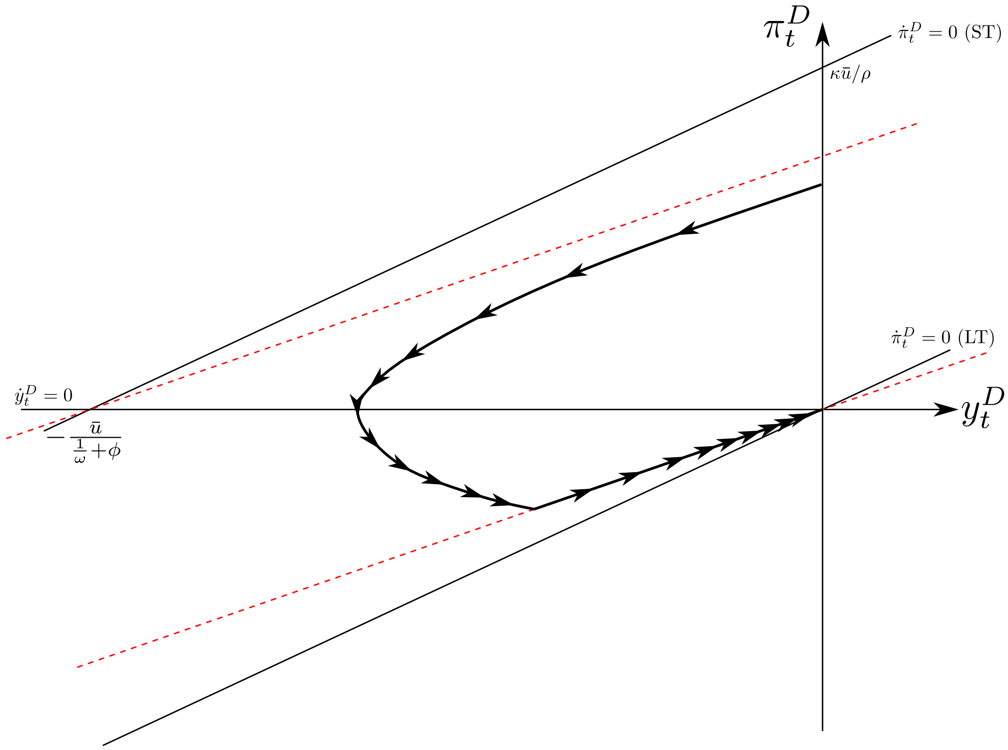


Figure 3: Output-inflation trade-off under free capital mobility.
*Note:* (ST) denotes short-term $\dot{\pi}_t^D = 0$ locus, (LT) denotes long-term $\dot{\pi}_t^D = 0$ locus.

support this path for the output gap differential, the terms of trade gap needs to follow a similar hump shape, indicating persistently (misaligned and) appreciated terms of trade throughout the episode.

In line with our discussion of Section 3.5, several patterns can prevail regarding cross-border capital flows. From (28), a hump-shaped trade deficit arises if $\eta > 1$, while a hump-shaped trade surplus arises if $\eta < 1$. When $\eta = 1$, trade remains balance in response to the cost-push shock.

## 4.2 Adjustment under constrained efficient regime

In the constrained efficient regime, the path of $\theta_t$ satisfies the targeting rule (23). Accounting for this fact when substituting the equilibrium terms of trade expression (19) into the NKPC in difference (18) again yields a dynamic equation for the cross-country difference in inflation as a function of itself and the cross-country difference in the output gap:

$$\dot{\pi}_t^D = \rho\pi_t^D - \kappa\left[\frac{1}{\omega} + \phi + \frac{2\alpha}{\chi\omega}\left[\chi - (1-2\alpha)\right]^2\right] y_t^D - \kappa u_t^D \tag{33}$$

where the last term in the large square bracket reflects the optimal management of the demand imbalance. This term is non-negative, and equal to zero only in the knife-edge case where $\chi = 1 - 2\alpha$. (33) and (31) now form the dynamical system in $\pi_t^D$ and $y_t^D$ whose solution represents the dynamics of the cross-country block of the model. Again, $\pi_t^D$ is a jump variable, and $y_t^D$ is predetermined at $y_0^D = 0$ under the optimal plan. The system is again saddle-path stable and is represented with a phase diagram in Figure 4.

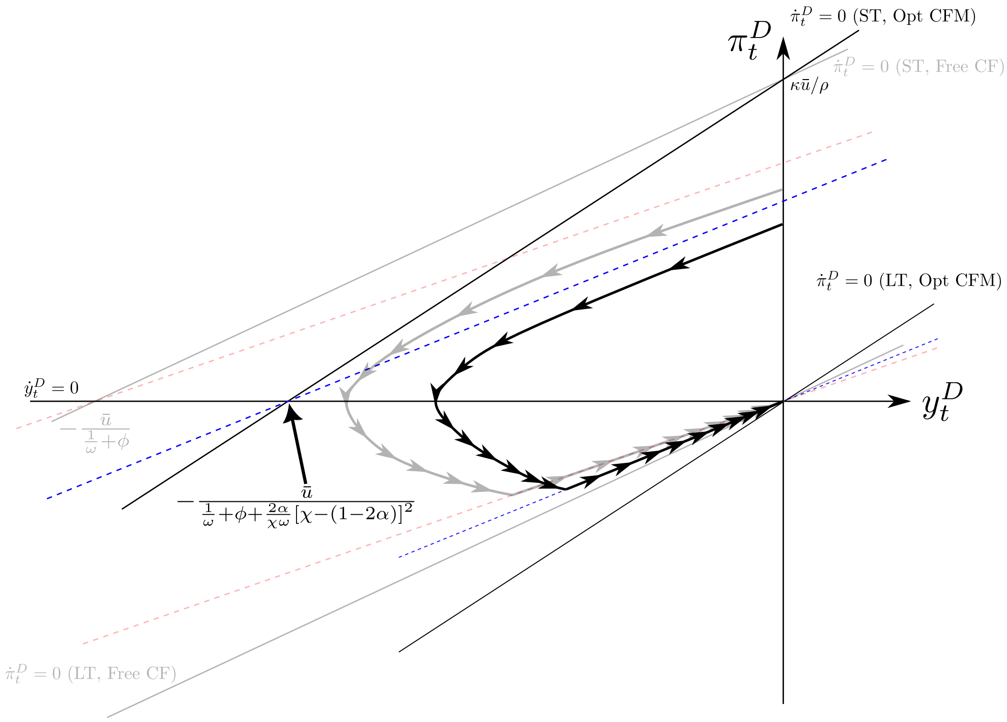


Figure 4: Output-inflation trade-off in the constrained efficient capital flow regime.
*Note:* (ST) denotes short-term $\dot{\pi}_t^D = 0$ locus, (LT) denotes long-term $\dot{\pi}_t^D = 0$ locus.

As under free capital mobility, the $\dot{y}_t^D = 0$ locus is described by $\pi_t^D = 0$. But this time, the $\dot{\pi}_t^D = 0$ locus is described by $\rho\pi_t^D = \kappa\left[\frac{1}{\omega} + \phi + \frac{2\alpha}{\chi\omega}\left(\chi - (1-2\alpha)\right)^2\right] y_t^D + \kappa u_t^D$. The only difference with the phase diagram of Figure 3 is that the $\dot{\pi}_t^D = 0$ locus now has a steeper slope of $\kappa\left[\frac{1}{\omega} + \phi + \frac{2\alpha}{\chi\omega}\left[\chi - (1-2\alpha)\right]^2\right]/\rho$ in the short-run. This slope is strictly steeper, except when $\chi = (1-2\alpha)$, in which case the two phase diagrams coincide. The phase diagram shows that the constrained efficient capital flow regime results in a more favorable trade-off between the stabilization of the cross-country difference in the output gap and the cross-country difference in domestic inflation, regardless of the direction of the inefficiency.

### 4.3 Quantitative Analysis

To further illustrate how the macroeconomic adjustments play out under both regimes, we now turn to presenting impulse response functions to an asymmetric cost-push shock in a calibrated version of the model. To do so, we draw heavily on the calibration used by Groll and Monacelli (2020) to study impulse responses to cost-push shocks. Rather than assuming a step function as in Sections 4.1 and 4.2, we consider a more standard mean reverting shock. In particular, we hit the economy with a cost-push shock of 10% that mean reverts at rate 0.42 per year, yielding an annual autocorrelation of 0.65, or equivalently, a quarterly autocorrelation of 0.9.

Table 1: Calibration

| Parameter | Description | Value/Target |
|---|---|---|
| $\rho$ | Discount factor | 0.04 |
| $\alpha$ | Degree of trade openness | 0.25 |
| $\varepsilon$ | Elasticity of substitution btw. differentiated goods | 7.66 |
| $\eta$ | Elasticity of substitution btw. Home and Foreign goods | 2 |
| $\chi$ | Trade elasticity | 3 |
| $\rho_\delta$ | Probability of being able to reset price | $1 - 0.75^4$ |
| $\rho_\mu$ | Persistence of cost-push shock in the Home | 0.65 |

The labor supply elasticity parameter $\phi$ is set to zero. The home bias parameter, $\alpha$, is set to 0.25, which implies a weight of 0.75 on domestically produced goods in the consumption basket. The trade elasticity $\chi$ plays an important role for our results, as it determines the direction of the inefficiency and the scope for topsy-turvy capital flows, with high elasticities making topsy-turvy capital flows more likely. Simonovska and Waugh (2014) report a range of trade elasticity estimates from 2.69 to 4.47. We conservatively set $\chi$ near the lower bound of this range to $\chi = 3$, which implies an elasticity of substitution between domestic and foreign good $\eta$ of 2. The discount rate parameter, $\rho$, and the parameter for the probability of adjustment of nominal prices, $\rho_\delta$, are both set to standard values: $\rho = 0.04$ and $\rho_\delta = 1 - 0.75^4$. Finally, the elasticity of substitution among differentiated intermediate goods, $\varepsilon$, is set to 7.66, corresponding to a 15% net markup. All parameters are hence set to the same value as in Groll and Monacelli (2020) (adjusting for our annual frequency).

Figure 5 illustrates the difference in the response of macroeconomic variables to the cost-push shock under free capital mobility vs. the constrained efficient capital flow regime.

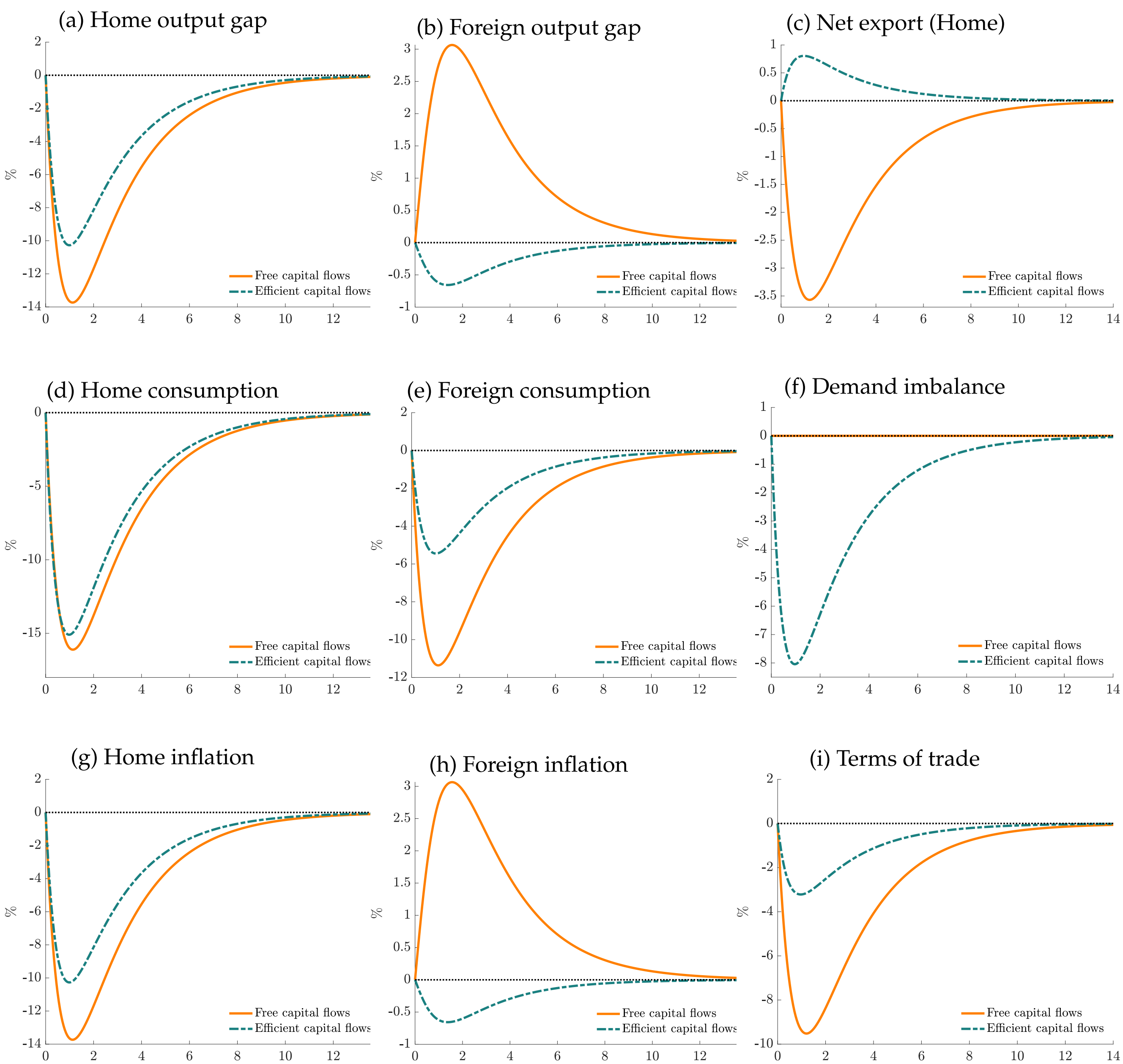


Figure 5: Impulse responses to cost-push shock in Home.
*Note:* Solid lines represent free capital mobility regime, and dashed lines represent constrained efficient capital flow regime.

It is well understood that under free capital mobility, the efficient allocation cannot be achieved following cost-push shocks. To limit PPI inflation in Home following the shock, monetary policy commits to negative output gaps in the future. The monetary policy response entails a terms of trade appreciation peaking at 9% and a positive spillover in Foreign, where the positive output gap nearly reaches 3%. Home runs a trade deficit of up to 3.5% of GDP.

In the optimal capital flow management regime, opening a demand imbalance in favor of Foreign helps reduce the magnitude of the Home output gap and mitigate international relative price misalignment at the expense of distorting the international risk-sharing condition. Home runs a modest trade surplus rather than a trade deficit, the Home output gap drops to -10% rather than -13% and the terms of trade appreciates by no more than 3%. The negative demand imbalance redirects demand toward Foreign, but the significantly smaller terms of trade appreciation results in a mildly negative Foreign output gap (rather than a positive one under free capital mobility). As a result, PPI inflation in Foreign is also more stable. Therefore, it is not a zero-sum game and both countries achieve a superior stabilization of output and inflation in the constrained efficient regime.

# 5 Discussion

In an effort to make our point as clear as possible, we have focused on the most basic specification of the open-economy New-Keynesian model. Enriching this basic model may create additional sources of externalities and inefficiencies, but as long as a wealth effect on labor supply is present and policy faces some output-inflation trade-off, our main insight regarding the poor functioning of international financial markets in a stagflation context can be expected to apply. In this section, we briefly discuss possible extensions of our analysis.

**Additional constraints on monetary policy.** To streamline the implications of output-inflation trade-offs for the normative properties of a free capital mobility regime, we purposely abstracted from additional constraints on monetary policy. These include a lack of commitment to future policies (i.e., discretionary policy), a lack of international cooperation (i.e., non-cooperative policy setting), and a lack of monetary independence in the two countries (such as resulting from a peg or a currency union). Such features would introduce extra constraints on stabilization policy, which capital inflows may contribute to loosen or tighten. Their presence would accordingly create distinct motives for financial market interventions, resulting in additional terms in the targeting rule for capital flow management (23).

**Alternative goods pricing specifications and deviations from the law of one price.** We assumed that export prices were sticky in producers' currency and that the law of one price held. Alternative assumptions regarding pricing currencies and deviations from the law of one price are known to place further constraints on monetary policy and accordingly give rise to more complex targeting rules than (21)-(22) (see Devereux and Engel, 2003 and Engel, 2011). Adopting these specifications would make other variables, such as the cross-country difference in consumer prices (also referred to as the average currency misalignment), relevant measures of the tightness of constraints on monetary policy, in addition to the output gap. As a result, they would also yield additional terms in the targeting rule for capital flow management (23), without invalidating our main insight.

**Non-cooperative capital flow management.** Our assumption that capital flow management policy is conducted cooperatively reflects our fundamental interest for understanding whether free capital mobility works to promote macroeconomic adjustment in a stagflation context *from the perspective of the world economy*. To study the related questions of whether individual countries face incentives to actively manage capital flows in such a context, assuming non-cooperative policy-making may be more appropriate. In this case, capital flow management by individual countries would trade off the dynamic terms of trade manipulation motives stressed by Costinot, Lorenzoni and Werning (2014) with the macroeconomic stabilization motives we have emphasized here.

# 6 Conclusion

We point to a macroeconomic externality operating via firms' marginal costs in open-economy models with nominal rigidities. For plausible values for the trade elasticity, this externality causes capital to flow in the wrong direction following shocks that create an output-inflation trade-off: while a constrained efficient regime would require outflows from the regions with the most depressed activity, a free capital mobility regime features capital inflows into such regions. Our results cast doubts on the classical view that free capital mobility promotes macroeconomic adjustment, in particular in a stagflation context.

Our analysis has implications beyond open-economy macroeconomics. Indeed, the insight that financial decisions may deteriorate policy trade-offs via externalities operating on the economy's supply side ought to apply more generally to other heterogeneous agents, multi-sector macroeconomic models with nominal rigidities. Given the rising popularity of heterogeneous agents New Keynesian (HANK) models and current concerns about the possibility of stagflation, the study of such externalities appears to be a pressing issue for future research.

# References

**Acharya, Sushant and Julien Bengui**, "Liquidity Traps, Capital Flows," *Journal of International Economics*, 2018, *114*, 276–298.

**Bengui, Julien**, "Macro-Prudential Policy Coordination," 2014. University of Montreal.

**Benigno, Gianluca and Pierpaolo Benigno**, "Price Stability in Open Economies," *Review of Economic Studies*, 2003, *70*, 743–764.

**_ , Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young**, "Financial Crises and Macro-Prudential Policies," *Journal of International Economics*, 2013, *89* (2), 453–470.

**_ , _ , _ , _ , and _** , "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective," *Journal of Monetary Economics*, 2016, *84*, 147–165.

**Benigno, Pierpaolo**, "Price Stability with Imperfect Financial Integration," *Journal of Money, Credit and Banking*, 2009, *40* (s1), 121–149.

**Bianchi, Javier**, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, December 2011, *101* (7), 3400–3426.

**_ and Enrique G Mendoza**, "Optimal time-consistent macroprudential policy," *Journal of Political Economy*, 2018, *126* (2), 588–634.

**_ and Louphou Coulibaly**, "Liquidity Traps, Prudential Policies, and International Spillovers," Working Paper 780, Federal Reserve Bank of Minneapolis July 2021.

**Caballero, Ricardo J. and Arvind Krishnamurthy**, "International and Domestic Collateral Constraints in a Model of Emerging Market Crises," *Journal of Monetary Economics*, 2001, *48* (1), 513–548.

**Calvo, Guillermo A.**, "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, September 1983, *12* (3), 383–398.

**Clarida, Richard, Jordi Gali, and Mark Gertler**, "A Simple Framework for International Mon- etary Policy Analysis," *Journal of Monetary Economics*, 2002, *49* (5), 879–904.

**Cole, Harold L. and Maurice Obstfeld**, "Commodity trade and international risk sharing : How much do financial markets matter?," *Journal of Monetary Economics*, August 1991, *28* (1), 3–24.

**Corsetti, Giancarlo and Paolo Pesenti**, "Welfare and macroeconomic interdependence," *Quarterly Journal of Economics*, March 2001, *116* (2), 421–445.

— **, Luca Dedola, and Sylvain Leduc**, "Chapter 16 - Optimal Monetary Policy in Open Economies," in Benjamin M. Friedman and Michael Woodford, eds., , Vol. 3 of *Handbook of Monetary Economics*, Elsevier, 2010, pp. 861–933.

— **,** — **, and** — , "Exchange Rate Misalignment, Capital Flows, and Optimal Monetary Policy Trade-Offs," CEPR Discussion Paper Working Paper DP12850 2018.

**Costinot, Arnaud, Guido Lorenzoni, and Ivan Werning**, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77 – 128.

**Coulibaly, Louphou**, "Monetary policy in sudden stop-prone economies," 2020. Working Paper, CIREQ.

**Devereux, Michael B. and Charles Engel**, "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility," *Review of Economic Studies*, 2003, *70* (4), 765–783.

**Engel, Charles**, "Currency Misalignments and Optimal Monetary Policy: A Reexamination," *American Economic Review*, 2011, *101*, 2796–2822.

**Farhi, Emmanuel and Ivan Werning**, "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates," NBER Working Paper 2012.

— **and** — , "Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows," *IMF Economic Review (Special Volume in Honor of Stanley Fischer)*, 2014, *62* (4).

— **and** — , "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 2016, *84* (5), 1645–1704.

— **and** — , "Fiscal Unions," *American Economic Review*, December 2017, *107* (12), 3788–3834.

**Fornaro, Luca and Federica Romei**, "The Paradox of Global Thrift," *American Economic Review*, 2019, *109* (11), 3745–3779.

**Geanakoplos, John D. and Heraklis M. Polemarchakis**, "Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete," in Walter P. Heller, Ross M. Star, and David A. Starrett, eds., *Uncertainty, Information and Communication*, Institute for International Economics, 1986.

**Greenwald, Bruce C. and Joseph E. Stiglitz**, "Externalities in Economies with Imperfect Information and Incomplete Markets," *Quarterly Journal of Economics*, 1986, *101* (2), 229–264.

**Groll, Dominik and Tommaso Monacelli**, "The inherent benefit of monetary unions," *Journal of Monetary Economics*, 2020, *111*, 63–79.

**Gromb, Denis and Dimitry Vayanos**, "Equilibrium and welfare in markets with financially constrained arbitrageurs," *Journal of Financial Economics*, 2002, *66* (3-2), 361–407.

**Jeanne, Olivier and Anton Korinek**, "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach," *American Economic Review*, May 2010, *100* (2), 403–07.

_ **and** _ , "Managing credit booms and busts: A Pigouvian taxation approach," *Journal of Monetary Economics*, 2019, *107*, 2–17.

_ **and** _ , "Macroprudential Regulation versus mopping up after the crash," *Review of Economic Studies*, 2020, *87*, 1470–1497.

**Korinek, Anton**, "Excessive Dollar Borrowing in Emerging Markets: Balance Sheet Effects and Macroeconomic Externalities," 2007. University of Maryland.

_ , "Regulating Capital Flows to Emerging Markets: An Externality View," *Journal of International Economics*, 2018, *111*, 61–80.

_ **and Alp Simsek**, "Liquidity Trap and Excessive Leverage," *American Economic Review*, 2016, *106* (3), 699–738.

**Lorenzoni, Guido**, "Inefficient Credit Booms," *Review of Economic Studies*, 2008, *75* (3), 809–833.

**Ottonello, Pablo**, "Optimal Exchange-Rate Policy Under Collateral Constraints and Wage Rigidity," *Journal of International Economics*, 2021, *131.*

**Schmitt-Grohe, Stephanie and Martin Uribe**, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

**Simonovska, Ina and Michael Waugh**, "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 2014, *92*, 34–50.

**Stiglitz, Joseph E.**, "The Inefficiency of the Stock Market Equilibrium," *Review of Economic Studies*, 1982, *49* (2), 241–261.

## APPENDIX TO "STAGFLATION AND TOPSY TURVY CAPITAL FLOWS"

# A Derivation of the loss function

The goal of the global planner is to maximize the average welfare function of Home and Foreign households. In this section, we rewrite the objective function in terms of the squared output gap, squared inflation and squared terms-of-trade and relative demand gap. Note that the period utility of the global planner is

$$v_t \equiv \frac{1}{2}\left[\log C_t - \frac{1}{1+\phi}(N_t)^{1+\phi}\right] + \frac{1}{2}\left[\log C_t^* - \frac{1}{1+\phi}(N_t^*)^{1+\phi}\right]$$

The loss relative to the efficient outcome is then $v_t - v_t^{max}$ where $v^{max}$ is the maximized welfare that is welfare when $C_t$, $C_t^*$, $N_t$ and $N_t^*$ take on their efficient values. We start by describing the efficient allocation then turn to deriving the second order approximation of the loss function.

**Efficient allocation.** The socially optimal allocation solves the following static problem

$$\max_{C_{H,t},C_{H,t}^*,C_{F,t},C_{F,t}^*,N_t,N_t^*} \frac{\eta}{\eta-1}\log\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right] - \frac{1}{1+\phi}(N_t)^{1+\phi}$$
$$+\frac{\eta}{\eta-1}\log\left[(1-\alpha)^{\frac{1}{\eta}}(C_{F,t}^*)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{H,t}^*)^{\frac{\eta-1}{\eta}}\right] - \frac{(N_t^*)^{1+\phi}}{1+\phi}$$

subject to

$$C_{H,t} + C_{H,t}^* = N_t \tag{A.1}$$
$$C_{F,t} + C_{F,t}^* = N_t^* \tag{A.2}$$

Let $\vartheta_{H,t}$ and $\vartheta_{F,t}$ denote the multipliers on (A.1) and (A.2). The first order conditions are

$$[C_{H,t}] :: \quad \vartheta_{H,t} = (1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{-\frac{1}{\eta}}(C_t)^{\frac{1}{\eta}-1} \tag{A.3a}$$
$$[C_{F,t}] :: \quad \vartheta_{F,t}^* = \alpha^{\frac{1}{\eta}}\,(C_{F,t})^{-\frac{1}{\eta}}\,(C_t)^{\frac{1}{\eta}-1} \tag{A.3b}$$
$$[C_{H,t}^*] :: \quad \vartheta_{H,t} = \alpha^{\frac{1}{\eta}}(C_{H,t}^*)^{-\frac{1}{\eta}}(C_t^*)^{\frac{1}{\eta}-1} \tag{A.4a}$$
$$[C_{F,t}^*] :: \quad \vartheta_{F,t}j = (1-\alpha)^{\frac{1}{\eta}}(C_{F,t}^*)^{-\frac{1}{\eta}}(C_t^*)^{\frac{1}{\eta}-1} \tag{A.4b}$$

$$[N_t] :: \quad (N_t)^{\phi} = \vartheta_{H,t} \tag{A.5a}$$
$$[N_t^*] :: \quad (N_t^*)^{\phi} = \vartheta_{F,t}^* \tag{A.5b}$$

Combining (A.3a) and (A.3b) after multiplying the first equation by $C^{H,t}\cdot$ and the second $C^{F,t}\cdot$ and proceeding similar with (A.4a) and (A.4b), we arrive to

$$\vartheta_{H,t}C_{H,t} + \vartheta_{F,t}^*C_{F,t} = 1 \tag{A.6a}$$
$$\vartheta_{H,t}C_{H,t}^* + \vartheta_{F,t}C_{F,t}^* = 1 \tag{A.6b}$$

Substituting the resource constraint into (A.5a) and (A.5b) yields $(N_t)^{1+\phi} + (N_t^*)^{1+\phi} = \vartheta_{H,t}(C_{H,t} + C_{H,t}^*) + \vartheta_{F,t}^*(C_{F,t} + C_{F,t}^*)$ which combined with (A.6a) and (A.6b) leads to

$$(N_t)^{1+\phi} + (N_t^*)^{1+\phi} = 2 \tag{A.7}$$

From market clearing and symmetry $\bar{C}_t = \bar{C}_t^* = \bar{N}_t = \bar{N}_t^* = 1$ where variables with a *bar* denote efficient values. It is also It is straightforward to see that $\bar{C}_{H,t} = \bar{C}_{F,t}^* = 1 - \alpha$ and $\bar{C}_{F,t} = \bar{C}_{H,t}^* = \alpha$. In log-deviations, we get

$$\bar{c}_{H,t} = \bar{c}_{H,t}^* = \bar{c}_{F,t} = \bar{c}_{F,t}^* = 0 \quad \text{and} \quad \bar{n}_t = \bar{n}_t^* = 0. \tag{A.8}$$

**Loss function.** The second order approximation of the period utility around the efficient allocation (using $\bar{N}^{1+\phi} = 1$ from (A.7) and symmetry) is given by

$$v_t = -\frac{1}{1+\phi} + \frac{1}{2}\Big[(c_t + c_t^*) - (n_t + n_t^*) - \frac{1+\phi}{2}\left((n_t)^2 + (n_t^*)^2\right) + o\left(||u||^3\right)\Big] \tag{A.9}$$

where $+o\left(||u||^3\right)$ indicate the $3^{rd}$ and higher order terms left out. Note from (A.8) and (A.9) that $v_t^{max} = -\frac{1}{1+\phi}$. The period loss function loss function is then

$$v - v_t^{max} = \frac{1}{2}\Big[(c_t + c_t^*) - (n_t + n_t^*) - \frac{1+\phi}{2}\left((n_t)^2 + (n_t^*)^2\right) + o\left(||u||^3\right)\Big] \tag{A.10}$$

We now need to use the second order approximation of the aggregate demand equations and aggregate employment to replace for $c_t$ and $n_t$. First note that after substituting for the international risk sharing condition (8), the aggregate demand for home goods can be rewritten as

$$Y_t = \left[(1-\alpha) + \alpha\left(S_t\right)^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\left[(1-\alpha) + \alpha\Theta_t^{-1}Q_t^{\eta-1}\right]C_t$$

Taking the second order approximation we get

$$y_t = c_t - \alpha\theta_t + \frac{\omega - (1-2\alpha)}{2}s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \frac{1}{2}\alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\eta-1)s_t\right]^2 + o(||u||^3) \tag{A.11}$$

where $\omega = \sigma\eta + (\sigma\eta - 1)(1-2\alpha)^2$. Similarly, demand for foreign good is given by

$$Y_t^* = \left[(1-\alpha) + \alpha\left(S_t\right)^{\eta-1}\right]^{\frac{\eta}{1-\eta}}\left[(1-\alpha) + \alpha\Theta_t Q_t^{1-\eta}\right]C_t^*,$$

and the second order approximation is given by

$$y_t^* = c_t^* + \alpha\theta_t - \frac{\omega - (1-2\alpha)}{2}s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \frac{1}{2}\alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\eta-1)s_t\right]^2 + o(||u||^3) \tag{A.12}$$

We then combine (A.11) and (A.12) to obtain

$$c_t + c_t^* = y_t + y_t^* + \alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\eta-1)s_t\right]^2 + o(||u||^3) \tag{A.13}$$

Using again (A.11) and (A.12) and after some algebraic manipulation we get

$$(c_t)^2 + (c_t^*)^2 = (y_t)^2 + (y_t^*)^2 - 2\alpha(1-\alpha)\left(\eta\right)^2\left(s_t\right)^2 \\ + 2\alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2 + o(||u||^3) \tag{A.14}$$

Aggregate employment is given $N_t = Y_t Z_t$ with $Z_t = \int_0^1 \left(P_{Ht(l)}/P_{Ht}\right)^{-\varepsilon} dl$. At the second order approximation $n_t = y_t + z_t + \frac{1}{2}y_t^2 + o(||u||^3)$ with $z_t = 0 + o(||u||^2)$. Thus, we have

$$n_t + n_t^* = y_t + y_t^* + \frac{1}{2}\left((y_t)^2 + (y_t^*)^2\right) + z_t + z_t^* + o(||u||^3) \tag{A.15}$$

$$(n_t)^2 + (n_t^*)^2 = (y_t)^2 + (y_t^*)^2 + o(||u||^3) \tag{A.16}$$

Plugging (A.13), (A.14), (A.15) and (A.16) into the (A.10) we obtain the following second order approximation of the period loss function

$$v - v_t^{max} = \frac{1}{2}\Big[z_t + z_t^* + (1+\phi)(y_t)^2 + (1+\phi)(y_t^*)^2 + 2\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + 2\alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2\Big] + o(||u||^3)$$

The objective of the global planner is to minimize the loss function is $\mathbb{L} = \int_0^\infty e^{-\rho t}$. Using

$$\int_0^\infty e^{-\rho t} z_t dt = \int_0^\infty e^{-\rho t} \text{var}_l\left(P_{H,t}(l)\right) dt = \frac{1}{\kappa}\int_0^\infty e^{-\rho t}\left(\pi_{H,t}\right)^2 dt$$
$$\int_0^\infty e^{-\rho t} z_t^* dt = \int_0^\infty e^{-\rho t} \text{var}_l\left(P_{F,t}^*(l)\right) dt = \frac{1}{\kappa}\int_0^\infty e^{-\rho t}\left(\pi_{F,t}^*\right)^2 dt$$

and our definition of world and difference variables $(\pi_{H,t})^2 + (\pi_{F,t}^*)^2 = 2[(\pi_t^W)^2 + (\pi_t^D)^2]$ and $(y_t)^2 + (y_t^*)^2 = 2[(y_t^W)^2 + (y_t^D)^2]$ we arrive to

$$\mathbb{L} = \frac{1}{2}\int_0^\infty e^{-\rho t}\Big[2\frac{\varepsilon}{\kappa}\left((\pi_t^W)^2 + (\pi_t^D)^2\right)^2 + 2(1+\phi)\left((y_t^W)^2 + (y_t^D)^2\right) + 2\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + 2\alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2\Big] \tag{A.17}$$

which corresponds to (20).

# B Optimal policy problem

We divide the loss (20) by a factor 2 since we can equivalently minimize a linear transformation of the objection function of the global planner. The optimal monetary policy problem is given by

$$\min_{\{\pi^W,\pi^D,x^W,y^D,s\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\Big[\frac{\varepsilon}{\kappa}\left((\pi_t^W)^2 + (\pi_t^D)^2\right) + (\sigma+\phi)\left((y_t^W)^2 + (y_t^D)^2\right) + \alpha(1-\alpha)(1-\sigma\eta)\eta(s_t)^2 + \sigma\alpha(1-\alpha)\left(\theta_t - (\sigma\eta-1)(1-2\alpha)\sigma^{-1}s_t\right)^2\Big]$$

subject to

$$\dot{\pi}_t^W = \rho\pi_t^W - \kappa(\sigma+\phi)y_t^W - \kappa u_t^W \tag{A.18}$$
$$\dot{\pi}_t^D = \rho\pi_t^D - \kappa(\sigma+\phi)y_t^D + \kappa\frac{\omega-1}{2}s_t - \kappa\alpha\sigma\theta_t - \kappa u_t^D \tag{A.19}$$
$$2y_t^D = \omega\sigma^{-1}s_t + (1-2\alpha)\theta_t \tag{A.20}$$

Letting $\varphi_t^W, \varphi_t^D$, be the co-state associated with (A.18), (A.19), the first order conditions are

$$\left[\pi_t^W\right] :: \quad \dot{\varphi}_t^W = -\frac{\varepsilon}{\kappa}\pi_t^W \tag{A.21}$$
$$\left[\pi_t^D\right] :: \quad \dot{\varphi}_t^D = -\frac{\varepsilon}{\kappa}\pi_t^D \tag{A.22}$$
$$\left[y_t^W\right] :: \quad 0 = -(\sigma+\phi)y_t^W + \kappa(\sigma+\phi)\varphi_t^W \tag{A.23}$$
$$\left[y_t^D\right] :: \quad 0 = -(\sigma+\phi)y_t^D + \kappa(\sigma+\phi)\varphi_t^D - \Lambda_t \tag{A.24}$$
$$[s_t] :: \quad 0 = -(\omega-1)y_t^D + \kappa(\omega-1)\varphi_t^D - \omega\sigma^{-1}\Lambda_t \tag{A.25}$$
$$[\theta_t] :: \quad 0 = -\sigma\alpha(1-\alpha)\theta_t + \frac{\omega-1}{4}(1-2\alpha)s_t + \kappa\alpha\sigma\varphi_t^D + \frac{1}{2}(1-2\alpha)\Lambda_t \tag{A.26}$$

together with the initial conditions $\varphi_0^j = 0$ and transversality conditions $\lim_{t\to\infty} e^{-\rho t}\varphi_t^j = 0$ for $j \in \{W, D\}$ and where $\Lambda_t$ is the Lagrange multiplier on (A.20).

## B.1 Proof of Proposition 1

Combining (A.24) and (A.25) we have $\Lambda_t = 0$. Substituting this back into (A.24), we obtain

$$y_t^D - \kappa\varphi_t^D = 0. \tag{A.27}$$

Differentiating (A.23) and (A.27) with respect to time and noting from (A.21) and (A.22) that $\kappa\dot{\varphi}_t^W = -\varepsilon\pi_t^W$ and $\kappa\dot{\varphi}_t^D = -\varepsilon\pi_t^D$, we obtain

$$\begin{aligned} \dot{y}_t^W + \varepsilon\pi_t^W &= 0 \\ \dot{y}_t^D + \varepsilon\pi_t^D &= 0 \end{aligned} \tag{A.28}$$

From (A.23), $y_t^W = \kappa\varphi_t^W$, and given that $\varphi_0^W = 0$, we have $y_0^W = 0$. From (A.27) and (A.20) we have $y_0^D = 0$ and $2y_0^D + \omega s_0 = 0$ which imply that $y_0^D = s_0 = 0$. Thus, integrating between 0 and $t$ we arrive to

$$y_t^W + \varepsilon(p_t^W - p_0^W) = 0 \tag{A.29}$$
$$y_t^D + \varepsilon(p_t^D - p_0^D) = 0 \tag{A.30}$$

## B.2 Proof of Corollary 1

We consider the targeting rule (A.28) for world variables and differentiate this rule to obtain $\ddot{y}_t^W + \varepsilon \dot{\pi}_t^W = 0$. We then use (A.18), $\dot{\pi}_t^W = \rho \pi_t^W - \kappa(\sigma + \phi) y_t^W - \kappa u_t^W$, to substitute for $\dot{\pi}_t^W$ and obtain

$$\ddot{y}_t^W - \rho \dot{y}_t^W - \varepsilon\kappa(1+\phi) y_t^W = \varepsilon\kappa u_t^W \tag{A.31}$$

The polynomial characteristic of this equation has one negative eigenvalue $z_1 < 0$ and one positive eigenvalue $z_2 > 0$ where

$$z_1 = \frac{1}{2}\left(\rho - \sqrt{\rho^2 + 4\kappa\varepsilon(1+\phi)}\right) < 0 \quad \text{and} \quad z_2 = \frac{1}{2}\left(\rho + \sqrt{\rho^2 + 4\kappa\varepsilon(1+\phi)}\right) > 0$$

The solution of this second order differential equation takes the form

$$y_t^W = \vartheta_0 e^{z_1 t} + \vartheta_1 \int_0^t e^{z_1(t-s)} u_s^W ds + \vartheta_2 \int_t^\infty e^{z_2(t-s)} u_s^W ds. \tag{A.32}$$

Differentiating (A.32) and relating each term to (A.31) we obtain

$$\vartheta_1 = \vartheta_2 = -\frac{\varepsilon\kappa}{z_2 - z_1}.$$

Next, from (A.32) for $t = 0$ we get

$$\vartheta_0 = y_0^W + \frac{\varepsilon\kappa}{z_2 - z_1} \int_0^\infty e^{-z_2 s} u_s^W ds$$

From the initial condition for the co-state variable $\varphi_0^W = 0$, the relation $y_t^W = \kappa \varphi_t^W$ implies that $y_t^W = 0$. The solution to the optimal monetary policy problem is thus

$$y_t^W = -\frac{\varepsilon\kappa}{z_2 - z_1}\left[e^{z_1 t} \int_0^t \left(e^{-z_1 s} - e^{-z_2 s}\right) u_s^W ds + \left(e^{z_2 t} - e^{z_1 t}\right) \int_t^\infty e^{-z_2 s} u_s^W ds\right]. \tag{A.33}$$

Using (A.28), the path for the world inflation under the optimal monetary policy satisfies

$$\pi_t^W = \frac{\kappa}{z_2 - z_1}\left[z_1 e^{z_1 t} \int_0^t \left(e^{-z_1 s} - e^{-z_2 s}\right) u_s^W ds + \left(z_2 e^{z_2 t} - z_1 e^{z_1 t}\right) \int_t^\infty e^{-z_2 s} u_s^W ds\right]. \tag{A.34}$$

From (A.33) and (A.34), it follows that the paths of the world variables $y_t^W$ and $\pi_t^W$ are independent of the path of $\theta_t$.

## B.3 Proof of Proposition 2

**Optimal capital flow management.** Notice again that by combining (A.24) and (A.25) we obtain $\Lambda_t = 0$. Substituting it into the optimality condition (A.26) we arrive to

$$
\begin{aligned}
2\alpha(1-\alpha)\theta_t &= (1-2\alpha)\frac{\omega-1}{2}s_t + 2\kappa\alpha\varphi_t^D \\
&= (1-2\alpha)\frac{\omega-1}{2}s_t + 2\alpha y_t^D
\end{aligned}
\tag{A.35}
$$

where the second equality uses (A.27). We then plug equation (A.20) into (A.35) to substitute for $y_t^D$. We get

$$
\begin{aligned}
2\alpha(1-\alpha)\theta_t &= (1-2\alpha)\frac{\omega-1}{2}\left[\frac{2}{\omega}y_t^D - \left(\frac{1-2\alpha}{\omega}\right)\theta_t\right] + 2\alpha y_t^D \\
&= \frac{\omega-(1-2\alpha)}{\omega}y_t^D - (1-2\alpha)^2\frac{\omega-1}{2\omega}\theta_t
\end{aligned}
\tag{A.36}
$$

Rearranging the expression (A.36) leads to

$$
\begin{aligned}
\frac{1}{2\omega}\left[\omega-(1-2\alpha)^2\right]\theta_t &= \frac{\omega-(1-2\alpha)}{\omega}y_t^D \\
\frac{\alpha}{\omega}\left[\chi\right]\theta_t &= \frac{2\alpha}{\omega}\left[\chi-(1-2\alpha)\right]y_t^D
\end{aligned}
\tag{A.37}
$$

where we use $\omega = 2\alpha\chi - (1-2\alpha)^2$ to obtain the second equality (A.37). Finally, we simplify the above expression (A.37) and arrive to

$$
\theta_t = \frac{\chi-(1-2\alpha)}{\chi}2y_t^D. \tag{A.38}
$$