Systemic Risk and Inefficient Debt Maturity

Julien Bengui*
Université de Montréal

September 2013

Abstract

This paper analyzes private debt maturity choices in a dynamic macroeconomic model in which financial frictions give rise to systemic risk in the form of amplification effects, and shows that decentralized maturity decisions may result in a socially excessive reliance on short-term debt. Long-term liabilities provide insurance against shocks to the asset side of the balance sheet, but they come at an extra cost. The debt maturity structure therefore maps into an allocation of aggregate risk between lenders and leveraged borrowers, and fundamental shocks propagate more powerfully in the economy when the maturity is shorter. The market equilibrium is not constrained efficient as borrowers fail to internalize their contribution to systemic risk and take on too much short-term debt in a decentralized economy. Macroprudential policy in the form of a tax on short-term debt can lead to Pareto improvements and result in less volatile allocations and asset prices.

Keywords: systemic risk, debt maturity, amplification effects, macroprudential regulation
JEL Codes: E44, D62, G28, G32, H23

*I am greatly indebted to Anton Korinek and Enrique Mendoza for their invaluable guidance. For useful suggestions and comments, I am also grateful to Sushant Acharya, Javier Bianchi, Sudipto Bhattacharya, Vito Gala, John Shea, and seminar participants at the 13th ECB-CFS Conference on Macroprudential Issues, the LACEA 2010 Annual Meeting, the 2011 AEA Annual Meeting, the Bank for International Settlements and the University of Maryland. All remaining errors are my own. Corresponding e-mail: `julien.bengui@umontreal.ca`.

1 Introduction

The recent global financial crisis has shown that liquidity problems, originally confined to a relatively small number of economic entities, can spread out rapidly. Through vicious spirals, they also lead to sudden losses of confidence in markets, causing massive asset price drops and cutbacks in bank lending. In the run-up to the crisis, highly leveraged entities, such as investment banks, hedge funds and off-balance-sheet vehicles, relied increasingly on very short-term liabilities to fund long-term assets. This trend is believed to have been a major factor behind the liquidity crunch that led to the unprecedented financial turmoil of 2008-2009 (Brunnermeier 2009). Was this widespread maturity mismatch just the efficient aggregate result of sound choices made by individually rational agents? Or was it in some sense *excessive*, in which case government intervention would have been warranted? This paper investigates this question by assessing within a quantitative theoretical framework the desirability of government policies that alter the debt maturity choice of leveraged economic agents.

In the wake of the recent crisis, academic economists, policymakers and observers have increasingly pushed for a broad reform of financial regulation. Central to the proposed reforms are macroprudential policies designed to limit behavior of market participants that tends to increase the whole financial system's vulnerability – so-called *systemic risk*. In addition to proposals to penalize high leverage and large institution sizes, most calls for new macroprudential regulations also suggest taxing large maturity mismatches.[1] But because of the general presumption that decentralized markets produce socially optimal outcomes through the "invisible hand," government interventions often need to be justified by the identification of a specific form of market failure. In our context, the market failure results from a "fire-sale externality" that causes excessive leverage and risk-taking by borrowers. Individual agents fail to internalize that by building up leverage and choosing a high risk exposure, they increase the likelihood of having to fire-sell assets beyond what would be socially desirable, thereby excessively depressing asset prices and tightening others' financing constraints in the event of adverse aggregate shocks.

This paper studies the debt maturity choice of leveraged agents in a formal framework where endogenous collateral constraints are a source of amplification of fundamental shocks. Long-term debt provides insurance against negative shocks to the value of assets held by leveraged borrowers, but it entails an extra cost over short-term debt because lenders need to be compensated for spending resources on enforcing long-term contracts. Borrowers choose their debt maturity by trading off the insurance benefits of long-term debt with its costs. But as they fail to internalize their contribution to systemic risk, they only consider the private insurance benefits of long-term debt and take on too little of it (i.e. too much short-term debt) in a decentralized market equilibrium. In such an environment, where the stability of leveraged borrowers' net worth has "public goods" properties, government intervention in the form of a tax on short-term debt can lead to Pareto improvements and result in less volatile allocations and asset prices.

We consider a model with two sets of agents and introduce financial frictions, as those play a key

[1] See, for instance, Brunnermeier, Crocket, Goodhart, Persaud, and Shin (2009).

role in the formal and informal analysis of systemic risk and macroprudential policy. The modeling framework builds on Kiyotaki and Moore (1997). Relatively patient agents (households) lend in equilibrium to less patient agents (entrepreneurs). Capital serves both as a factor of production and as collateral for loans. When the only type of claims that agents can trade is one-period non-state-contingent bonds, entrepreneurs are naturally more exposed to aggregate risk (productivity shocks) than households because of leverage. A negative shock disproportionately hurts the net worth of entrepreneurs, leading them to reduce borrowing and fire-sell assets. As in Kiyotaki and Moore (1997), fundamental shocks get amplified as these fire-sales lead to a further decline in asset prices and net worth, causing yet another round of deleveraging. By letting agents trade long-term bonds alongside usual one-period non-state-contingent bonds, we allow for the possibility of better risk-sharing between households and entrepreneurs. Even though long-term bonds are a promise to non-state-contingent payments, their one-period return is state-contingent since the market price of the future payment stream generally depends on aggregate conditions, as in Angeletos (2002) and Buera and Nicolini (2004). Since the prices of both long-term bonds and physical assets are pro-cyclical, the issuance of long-term debt by borrowers (entrepreneurs) effectively shifts risk towards lenders (households). A longer debt maturity structure thus translates into a lower relative risk exposure of leveraged entrepreneurs. When adverse shocks hit, the value of the entrepreneurs' assets shrinks, but so does the value of their liabilities, which mitigates the effect on their net worth. By reducing leveraged entrepreneurs' net worth's sensitivity to fundamental shocks, a longer debt maturity structure also reduces the scope of financial amplification in the economy, resulting in less volatile allocations and asset prices.

In this environment where a shorter debt maturity maps into more volatile aggregate economic variables, we ask whether debt maturity choices made by individually rational agents result in socially efficient risk allocations. Short-term debt is cheaper than long-term debt, because in the model enforcing long-term contracts is costly. In choosing their debt maturity, entrepreneurs hence trade-off the private insurance benefits of long-term debt with the cost advantage of short-term debt. But since (1) lower net worth causes fire-sales, (2) fire-sales depress the price of capital, and (3) the price of capital matters for other entrepreneurs' borrowing capacity, the social insurance benefits of long-term debt outweigh its private benefits. As a result of this pecuniary externality in an incomplete market setting, entrepreneurs issue too much short-term debt and too little long-term debt in a decentralized equilibrium. We show that a constant tax on short-term debt can lead to Pareto improvements and less volatile aggregate economic variables by inducing entrepreneurs to rely on longer-term funding. In fact, in our model entrepreneurs are made better-off even when the proceeds of the tax are wasted in unproductive expenditures instead of being rebated lump-sum.

The paper is related to several strands of the literature. First, it relates to a broad theoretical literature in corporate finance and banking that analyzes debt maturity choice in partial equilibrium. For the most part, this literature attempts to rationalize the empirical prevalence of short-term debt in the financial and non-financial corporate sector. Flannery (1986) and Diamond (1991) argue that short-term debt issuance can act as a signaling device in frameworks with asymmetric information between borrowers and lender. Diamond and Dybvig (1983) rationalize demandable debt as an efficient mechanism to deal with depositors' exposure to liquidity shocks, while Calomiris and Kahn (1991) and Diamond and Rajan (2001) emphasize the incentive roles of short-term debt in environments with moral hazard. In contrast to this literature, the present paper stresses undesirable aspects of short-term debt and argues that too much of it may be issued in decentralized markets.

The second literature to which this paper relates is the macroeconomic literature on financial amplification or financial accelerator effects, which analyzes the role of financial frictions in general equilibrium. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) show that in the presence of financial frictions, endogenous variations in borrowers' net worth can lead to amplification of fundamental economic shocks. The formal modeling framework adopted in this paper shares several aspects of the quantitative theoretical implementations of these ideas by Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Iacoviello (2005), Mendoza and Smith (2006) and others. On the normative side, Lorenzoni (2008) and Bianchi (2011) find that individual agents may overborrow as they do not internalize the tightening of financing conditions they impose on other agents through their subsequent deleveraging in the event of bad shocks. Korinek (2009a) finds that atomistic agents in emerging markets may rely excessively on dollar debt, as they do not internalize the pressure put on the exchange rate through their cut in aggregate demand during financial crises, and Korinek (2009b) argues more generally that agents choose excessively risky financial structures in the presence of financial accelerator effects. This paper complements this body of research by showing that the debt maturity chosen by constrained borrowers in a decentralized equilibrium can be socially inefficient and can result in excessively volatile allocations and asset prices.

Finally, the paper also relates to a literature that analyzes the maturity structure of capital flows to emerging markets. Paralleling the results of the corporate finance papers mentioned above, Rodrick and Velasco (1999), Tirole (2003) and Jeanne (2009) argue that short-term debt can act as a disciplining device for opportunistic sovereign borrowers, although the latter recognize that under some circumstances, short-term debt accumulation by private agents can be socially excessive. Broner, Lorenzoni, and Schmukler (2008) explain emerging market governments' reliance on short-term debt by appealing to international lenders' risk aversion and fluctuations thereof. Like most of the corporate finance literature, the analysis in these papers is based on heavily stylized 3-period partial equilibrium models. In contrast, the present paper studies the positive and normative implications of debt maturity choices in a tractable infinite horizon dynamic stochastic general equilibrium framework with risk averse borrowers and lenders.

The environment is presented in Section 2 and the competitive equilibrium is defined and characterized in Section 3. Section 4 discusses the rationale for macroprudential policy. Section 5 presents the quantitative results and Section 6 concludes.

2 The model

We consider an environment, inspired by Kiyotaki and Moore (1997), with two sets of agents - households and entrepreneurs - and one source of (aggregate) risk. Both types of agents are risk-averse consumers and derive benefits from a physical asset. Entrepreneurs, who for modeling purposes are assumed to be less patient, borrow from households in equilibrium, and produce the consumption good out of the physical asset and labor using a constant returns to scale technology. Households supply labor and savings to the entrepreneurial sector, and use the physical asset for home production. Financial markets are both imperfect and incomplete. In addition to an enforcement friction that underlies a collateral constraint faced by borrowers, asset markets are exogenously assumed to be incomplete in that agents are only allowed to trade short-term and long-term non-state-contingent bonds. Short-term bonds are one-period non-state-contingent bonds, while long-term bonds are modeled as a perpetuity. Note that although the cash flows attached to a long-term bond are non-state-contingent, the one-period rate of return on this bond is state-contingent, as the price of long-term bonds generally varies with economic conditions. The presence of long-term bonds therefore creates risk-sharing opportunities in the economy by enabling agents to form bond portfolios with state-contingent returns. There are two goods: a consumption good and a capital good. The consumption good is perishable, while capital is in fixed supply and does not depreciate.

Households. There is a unit mass of identical infinitely-lived households in the economy. Each household maximizes expected lifetime utility, given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - G(L_t) \right]$$

where E_0 is the expectation operator conditional on period 0 information, β is a discount factor, $u(\cdot)$ is a constant-relative-risk-aversion (CRRA) period utility function for consumption, $G(\cdot)$ is an increasing and convex labor disutility function, and C_t, K_t, L_t denote period t consumption, physical asset holding and labor supplied, respectively. Note that the stock of physical assets relevant for home production in period t is the one carried into period $t+1$. This amounts to assuming that physical assets are traded *cum-dividend*. Households can also choose to invest in short-term bonds and long-term bonds. q_t^S denotes the price of the short-term (discount) bond. Similarly, q_t^L denotes the price of the long-term bond, which entitles its holder to payments of one unit of the consumption good in every future period until infinity. The return on holding short-term bonds between t and $t+1$ is known in t, while the one-period return on holding long-term bonds is state-dependent, because the $t+1$ market value of the remaining payment stream of a perpetuity depends on the state of the economy. A representative household chooses sequences of consumption, capital, short-term bonds and long-term bonds to maximize expected lifetime utility subject to the following period budget constraint:

$$C_t + q_t K_{t+1} + q_t^S B_{t+1}^S + q_t^L B_{t+1}^L = A_t F(K_{t+1}) + w_t L_t + q_t K_t + B_t^S + (1 + q_t^L) B_t^L - \vartheta B_t^L,$$

where $F(\cdot)$ is an increasing and concave home production function, q_t is the price of capital, w_t is the wage rate, B_t^S is the household's holding of short-term bonds, B_t^L is its holding of long-term bonds, and ϑ represents a monitoring cost which long-term bond holder have to incur to prevent borrowers from absconding with the remaining stream of payments due. [2] Further, as monitoring activities generally result in unproductive expenditures, we assume that the monitoring cost is a resource cost. Consequently, short-term debt contracts have a relative advantage over long-term debt contracts in that they avoid the need for unproductive monitoring activities. From a social perspective, this implies a trade-off between the insurance (risk-sharing) benefits of long-term debt and the cost advantage of short-term debt.[3] Section 3.2 offers a detailed discussion of individual agents' bond/debt maturity choice in the model.

Household behavior is characterized by the following four optimality conditions:

$$w_t = \frac{G'(L_t)}{u'(C_t)}, \tag{1}$$

$$q_t u'(C_t) = A_t F'(K_{t+1}) u'(C_t) + \beta E_t \left[q_{t+1} u'(C_{t+1}) \right], \tag{2}$$

$$u'(C_t) = \beta \frac{1}{q_t^S} E_t \left[u'(C_{t+1}) \right], \tag{3}$$

$$u'(C_t) = \beta E_t \left[\frac{1 + q_{t+1}^L}{q_t^L} u'(C_{t+1}) \right] - \beta E_t \left[\frac{\vartheta}{q_t^L} u'(C_{t+1}) \right]. \tag{4}$$

(1) describes optimal labor supply, while (2), (3) and (4) are standard Euler equations characterizing the household's optimal holdings of the physical asset, short-term bonds and long-term bonds.

Entrepreneurs. There is a continuum of mass one of identical entrepreneurs with infinite horizon. Entrepreneurs consume and operate a technology which produces consumption goods out of physical capital and labor inputs. To make entrepreneurs borrow in equilibrium, we assume that they discount the future more strongly than households. Each entrepreneur faces a collateral constraint which limits the value of his total debt to a fraction of the value of the capital he holds.[4] As developed below, this collateral constraint can be interpreted as an incentive compatibility constraint in an environment with enforcement frictions.[5] The lower discount factor ensures

[2] To preserve a meaningful bond portfolio choice even as the stochastic noise in the model approaches zero, we assume that this cost is of second-order, i.e. that $\vartheta \equiv (\epsilon_t^{\vartheta})^2$, where ϵ_t^{ϑ} is a zero-mean i.i.d. random variable with variance σ_{ϑ}^2 of the same order of magnitude as the variance of shocks to TFP. The assumption that the monitoring cost is stochastic is made for technical reasons only and affects the equilibrium dynamics only at third- and higher orders. Because of the presence of portfolio choice, we solve the model by approximating the decision rules around a deterministic steady state to which the allocations and prices converge when the scale of the stochasticity become arbitrary small. If the monitoring cost was a constant independent of that scale, the bond portfolio choice would be trivial in the limit: as the insurance benefits of long-term debt depend on the scale of TFP shocks but the monitoring costs are kept fixed, for arbitrarily small TFP shocks, short-term contracts would strictly dominate long-term contracts. By making monitoring costs stochastic, we are able to maintain a non-trivial trade-off between short- and long-term debt, even as the scale of the stochastic noise approaches zero, through effectively stabilizing the relationship between the costs and benefits of long-term debt.

[3] Kiyotaki and Moore (2003, 2005) consider a setup where borrowers need to pay a deadweight securitization cost to ensure the future liquidity (resellability) of long-term claims, putting forward the argument that multilateral commitment to repay debt is generally more demanding than bilateral commitment (the later being the only type of commitment relevant for short-term debt issuance). This securitization cost increases the cost of borrowing long-term. Our monitoring cost ϑ puts the burden of costly contract enforcement on the lender but effectively achieves the same effect.

[4] Given a lower discount factor, in the absence of limits on their borrowing, entrepreneurs would accumulate debt to a point where their long-run consumption would converge towards zero.

[5] Note that in principle, households are subject to the same enforcement friction as entrepreneurs and should

that entrepreneurs remain financially constrained in the long run. Each entrepreneur maximizes expected lifetime utility, given by:

$$E_0 \sum_{t=0}^{\infty} \gamma^t u(c_t),$$

where γ is a discount factor satisfying $\gamma < \beta$, and c_t denotes period t consumption. The entrepreneur's period budget constraint is given by

$$c_t + q_t k_{t+1} + q_t^S b_{t+1}^S + q_t^L b_{t+1}^L = A_t f(k_{t+t}, l_t) - w_t l_t + q_t k_t + b_t^S + (1 + q_t^L) b_t^L,$$

where k_t is the entrepreneur's holding of physical capital, l_t is the hired labor, b_t^S is his holding of short-term bonds, b_t^L is his holding of long-term bonds, and $f(\cdot)$ is a constant returns to scale production function.[6]

Entrepreneurs also face a sequence of collateral constraints, given by

$$-q_t^S b_{t+1}^S - q_t^L b_{t+1}^L \leq \kappa q_t k_{t+1}. \tag{5}$$

This constraint limits the total value of outstanding debt to a fraction of the value of capital held by a borrower. It is akin to the collateral constraints in Aiyagari and Gertler (1999) and Kiyotaki and Moore (1997), and as in these papers it has the potential to generate financial amplification effects through the impact of asset price changes on agents' borrowing capacity. We interpret the constraint as arising from a limited enforcement problem, but refrain from modeling its microeconomic foundation explicitly.

A representative entrepreneur chooses sequences of consumption, capital, short-term bonds and long-term bonds to maximize expected life-time utility subject to a sequence of period budget constraints and collateral constraints. Optimal behavior by entrepreneurs is characterized by the following four conditions

$$A_t f_l(k_{t+1}, l_t) = w_t \tag{6}$$

$$q_t u'(c_t) = A_t f_k(k_{t+1}, l_t) u'(c_t) + \gamma E_t \left[q_{t+1} u'(c_{t+1}) \right] + \kappa \mu_t q_t \tag{7}$$

$$u'(c_t) = \gamma \frac{1}{q_t^S} E_t \left[u'(c_{t+1}) \right] + \mu_t, \tag{8}$$

$$u'(c_t) = \gamma E_t \left[\frac{1 + q_{t+1}^L}{q_t^L} u'(c_{t+1}) \right] + \mu_t, \tag{9}$$

where μ_t is the non-negative multiplier on the collateral constraint, and by the complementary slackness condition

$$\mu_t \left(q_t^S b_{t+1}^S + q_t^L b_{t+1}^L + \kappa q_t k_{t+1} \right) = 0. \tag{10}$$

therefore face the same collateral constraint. However, given their higher discount factor, households will turn out to be lenders and not borrowers in equilibrium. Imposing a borrowing constraint in their decision problem would thus be superfluous.

[6] We omit the monitoring cost in the budget constraint, because entrepreneurs will end up being issuers of long-term bonds in equilibrium ($b_t^L < 0$), and monitoring costs are borne by lenders (households).

(6) describes entrepreneurs' labor demand, while (7), (8) and (9) are conventional Euler equations for capital, short-term bonds and long-term bonds. From (8) and (9) we see that the borrowing constraint can induce a wedge between the current marginal value of wealth and the discounted expected value of next period's marginal value of wealth. It is also apparent from (7) that when the collateral constraint binds, entrepreneurs value capital more highly as it helps relax the constraint.

Fundamentals. TFP is assumed to follow a first-order autoregressive process

$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_t^A,$$

where ϵ_t^A is an i.i.d. random variable with variance σ_A^2.[7]

3 Competitive Equilibrium

It is convenient to define the variables $B_t \equiv q_{t-1}^S B_t^S + q_{t-1}^L B_t^L$, $\Phi_t \equiv q_{t-1}^L B_t^L$, $b_t \equiv q_{t-1}^S b_t^S + q_{t-1}^L b_t^L$ and $\phi_t \equiv q_{t-1}^L b_t^L$. A competitive equilibrium of the model can then be defined by sequences of state-contingent allocations $\{c_t, C_t, l_t, L_t, b_{t+1}, B_{t+1}, \Phi_{t+1}, \phi_{t+1}, k_{t+1}, K_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, q_t^S, q_t^L, q_t\}_{t=0}^{\infty}$ such that: (a) households maximize expected lifetime utility subject to their sequence of budget constraints, taking as given prices and initial conditions (B_0, Φ_0, K_0), (b) entrepreneurs maximize expected lifetime utility subject to their sequence of budget and collateral constraints, taking as given prices and initial conditions (b_0, ϕ_0, k_0), and (c) the markets for labor, short-term bonds, long-term bonds and capital clear[8]:

$$\begin{aligned} L_t &= l_t, \\ b_{t+1} - \phi_{t+1} + B_{t+1} - \Phi_{t+1} &= 0, \\ \phi_{t+1} + \Phi_{t+1} &= 0, \\ k_{t+1} + K_{t+1} &= \bar{K}, \end{aligned}$$

where $\bar{K}$ is the fixed supply of capital in the economy.

We quantitatively analyze the dynamics of the model in the neighborhood of a deterministic steady state to which competitive equilibrium allocations and prices converge when the scale of the stochastic noise in the model becomes arbitrarily small. Even though the maturity structure is not uniquely determined in the deterministic steady state, as short- and long-term bonds are perfect substitutes in the absence of stochastic shocks, the other "non-portfolio" variables are uniquely pinned down. By focusing our attention on the local dynamics of the model, we assume that the entrepreneur's collateral constraint always bind.[9]

[7] At a technical level and for the purpose of solving the model, a second source of uncertainty in the economy arises from monitoring costs. For the purpose of solving the model, we assume that ϵ_t^ϑ is an i.i.d. random variable with variance σ_ϑ^2. Given the retained formulation, up to a second-order of accuracy the realizations of this shock have no effect on the equilibrium dynamics of the model's variables. The presence of the monitoring cost (but not the realizations of ϵ_t^ϑ) is nonetheless a key determinant of debt/bond maturity choices.

[8] Goods market clearing then follows from Walras' law.

[9] In simulations, we find that the shadow price of this constraint remains positive in each of the 100,000 periods.

3.1 Deterministic steady state

We consider a deterministic steady state in which long-term debt enforcement is costless ($\vartheta = 0$). From the household's Euler equations for short term and long term bonds, the bond prices are given by $q^S = \beta$ and $q^L = \beta/(1-\beta)$. The gross interest rates on these two bonds are thus equal and given by $r^S = r^L = \beta^{-1}$. Given that the two bonds have the same deterministic returns in the steady state, they are indistinguishable. This illustrates why the agents' debt portfolios are not uniquely pinned down in the absence of stochastic shocks and monitoring costs. From the entrepreneur's Euler equations (8) or (9), one gets $\mu = (1-\gamma/\beta)u'(c) > 0$, meaning that the entrepreneurs are constrained in the deterministic steady state. Combining the two agents' Euler equations for capital, (2) and (7), we can write

$$AF'(K) = \frac{\beta(1-\beta)}{\beta(1-\beta)+(\beta-\kappa)(\beta-\gamma)} Af_k(k,l) \tag{11}$$

This expression illustrates that as long as $\kappa \neq \beta$, capital is allocated inefficiently in the deterministic steady-state. In the realistic case where $\kappa < \beta$, the marginal product of capital is higher in the entrepreneurial sector than in the household sector.

3.2 Costs and benefits of long-term debt

The central point of the paper is to show that debt maturity choices made by individually rational agents in an environment where financial frictions give rise to amplification effects are not necessarily efficient, and that when they are not, debt contracts can have an excessively short maturity. It is thus worth clarifying the precise elements that affect the agents' debt maturity choices in the model.

Using the household's Euler equations, we can express the prices of capital and long-term bonds as

$$q_t = E_t \sum_{j=0}^{\infty} \frac{\beta^j A_{t+j} F'(K_{t+1+j}) u'(C_{t+j})}{u'(C_t)},$$

and

$$q_t^L = E_t \sum_{j=1}^{\infty} \frac{\beta^j u'(C_{t+j})}{u'(C_t)}^{j-1} (1-\vartheta).$$

From these expressions, it can be recognized that fluctuations in households' consumption resulting from aggregate shocks will be associated with co-movements in the prices of capital and long-term bonds due to fluctuations in the common stochastic discount factor. A positive TFP shock will lead to higher current household consumption (via higher wages and higher home production), lower current household marginal utility of consumption, and thus in equilibrium to higher prices of capital and long-term bonds.[10] At the same time, leveraged entrepreneurs who fund part of their capital holdings with debt are highly exposed to aggregate shocks: not only do they suffer from less

[10]The strength of the price responses will depend, among other things, on how much households are exposed to aggregate shocks, but its direction will be unambiguous.

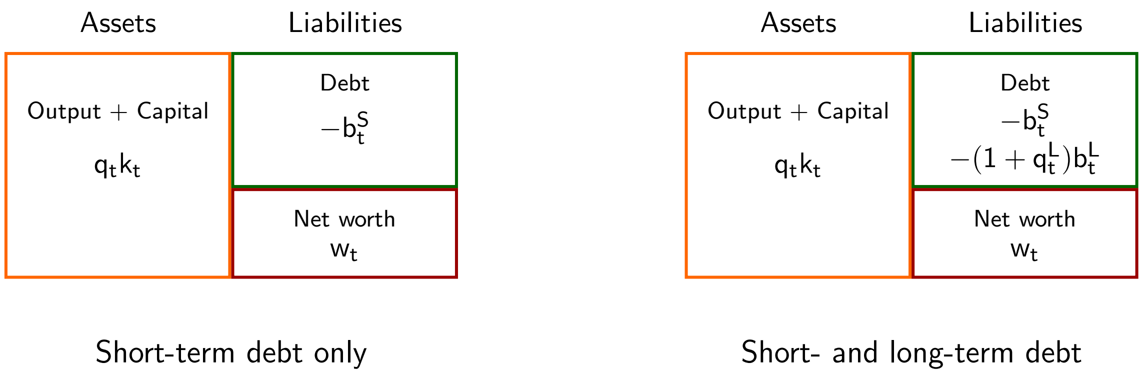

Figure 1: Balance sheets of leveraged entrepreneur.

productive capital when a bad shock hits, but they are also hurt by the asset price drop associated with the scarcity of current resources. Figure 1 represents stylized balance sheets of leveraged borrowers (entrepreneurs) in the cases with and without long-term debt. When only short-term debt is available, the market value of the debt is predetermined, while the value of the assets is state contingent. This results in a high sensitivity of entrepreneurs' net worth to aggregate shocks. When long-term debt is also available, the value of the debt is state-contingent, since the market price of the future payment stream attached to long-term debt depends on current conditions. In particular, the market value of the debt rises in good times and shrinks in bad times. Issuing long-term debt thus provides entrepreneurs with a hedge against fluctuations in the value of their assets. Equivalently, it allows entrepreneurs to pass on to households some of the risk to which they are naturally exposed.

To which extent will private agents make use of the risk-sharing vehicle provided by long-term debt contracts? This will depend on the premium on long-term debt charged by lenders. In the absence of costly enforcement of long-term contracts (when $\vartheta = 0$), this premium will correspond to a pure term premium: since the return on long-term bonds is positively correlated with lenders' consumption in equilibrium, lenders will demand a compensation for holding these bonds. Borrowers will then trade-off the insurance benefits of long-term debt with its extra cost, and choose a debt maturity such that those two are equalized. When the enforcement of long-term contracts is costly ($\vartheta > 0$), lenders will require a compensation for holding long-term bonds beyond what can be attributed to a pure term premium. In effect, lenders will pass on the burden of costly enforcement to borrowers, since in equilibrium they need to be indifferent at the margin between saving in short- and long-term bonds. Faced with this higher cost of long-term debt, borrowers will generally choose debt maturity structures shorter than what they would choose in the absence of costly enforcement of long-term contracts. From a social perspective, short-term debt has an advantage in that it avoids the waste of resources in unproductive monitoring activities. Our model thus captures in a reduced form the incentive benefits of short-term debt put forward by Calomiris and Kahn (1991) and others. A contribution of our paper is then to show that the market can fail to produce allocations that efficiently balance the trade-off between the risk-sharing benefits of long-term debt and the cost advantage of short-term debt.

3.3 Analytical results

It turns out that when enforcement of long-term debt is costless ($\vartheta = 0$), the availability of short- and long-term bonds results in competitive equilibrium allocations that achieve an efficient degree of risk-sharing. This is because in spite of borrowing constraints, short- and long-term bonds can provide agents with a very valuable hedge. In the absence of bonds with state-contingent returns, entrepreneurs are more exposed to risk than households because of their leveraged positions: they finance part of their capital holdings with debt and therefore tend to suffer more in the event of a negative TFP shock (which depresses the price of the capital). But given the state-contingent nature of the returns on long-term bonds, entrepreneurs can insure against fluctuations in the value of their capital by going short on long-term bonds, i.e. by issuing long-term debt, while going long on short-term bonds. Indeed, under some conditions, this opportunity results in a fully efficient allocation of risk between households and entrepreneurs, which leads to the following proposition.

Proposition 1. *When agents have log utility, TFP is serially uncorrelated and enforcement of long-term debt is costless (i.e. when $\vartheta = 0$), risk markets are effectively complete. As a result, aggregate risk is shared equally by households and entrepreneurs, and the wealth distribution is time-invariant. Furthermore, the entrepreneur's collateral constraint always binds, the capital allocation is fixed at its deterministic steady state value (i.e. fire-sales never occur) and the economy displays no persistence.*

Proof. See appendix B. □

The intuition for the effective completeness of risk markets comes from the fact that when long-term debt enforcement is costless, under log utility and serially uncorrelated TFP, the capital and bond prices are all linear functions of the single state variable (TFP). It is therefore possible to construct a bond portfolio whose fluctuations in value match exactly the changes in the relative wealth distribution caused by fluctuations in TFP under asymmetric holdings of capital. The result is similar to the ones in Angeletos (2002) and Buera and Nicolini (2004), who find that a government can use non-contingent debt of different maturities to achieve complete markets Ramsey allocations. Our result requires a more stringent restriction on preferences than theirs, but puts less demand on asset markets: complete markets in these papers generally requires debt instruments of as many maturities as possible states of nature, while our result holds for an arbitrarily large number of states and just two maturities.

We view the equal sharing of aggregate risk as an efficient outcome, given identical risk tolerances and the CRRA property of the log utility function. The constant relative risk aversion makes the desired risk-exposure of agents independent of their wealth levels, so it is socially desirable to let households and entrepreneurs share aggregate risk equally, even though the former are wealthier than the later. A remarkable aspect of proposition 1 is that perfect risk sharing is achieved despite the presence of a collateral constraint, which a priori puts restrictions on the trade of the available financial claims.

The *no fire-sales* outcome also represents a strong result. In the presence of short-term debt only, as in the generic model of Kiyotaki and Moore (1997), aggregate shocks relax or tighten leveraged borrowers' constraints, and result in them increasing or decreasing their capital holdings, thereby setting in motion a financial amplification mechanism. Under the conditions of proposition 1, aggregate shocks do indeed relax or tighten the collateral constraint of borrowers, and thus affect their demand for the physical asset, but it happens to do so in exactly the same proportions as the wealth effect on the unconstrained agents' demand for the asset. In equilibrium, the asset price adjusts to induce agents to demand a constant amount of capital, and there is no transfer of asset between the constrained and unconstrained sectors. The price of capital appreciates following a good shock and depreciates following a bad shock, but the allocation of capital never deviates from its deterministic steady state value. The fact that fire-sales never occur in equilibrium explains the efficiency of risk allocations. In a similar environment, Korinek (2009b) finds that risk allocations can fail to be constrained efficient even when agents have access to a full set of state-contingent assets. There, the inefficiency derives from the fact that agents undervalue wealth in states of nature where fire-sales depress asset prices. Under less restrictive assumptions than the ones in proposition 1, fire sales will occur in our model and risk allocations will generally fail to be efficient.

But under the assumptions of proposition 1, the competitive equilibrium prices and allocations take particularly simple forms. Define the variable $Y_t \equiv [f(k_{t+1}, l_t) + F(K_{t+1})]$, such that aggregate output in period t is given by $Y_t A_t$. Capital and labor are always allocated as in the deterministic steady state: $k_{t+1} = k$, $K_{t+1} = K$, $l_t = l$ for all t. Households and entrepreneurs consume a constant fraction of aggregate output every period: $C_t = (1-\omega)YA_t$ and $c_t = \omega Y A_t$, where ω is related to the relative wealth positions.[11] The price of capital is $q_t = \frac{1}{1-\beta}F'(K)A_t$. Assuming normality of the innovations to log TFP, the prices of short-term bonds and long-term bonds are given by

$$q_t^S = \beta e^{\frac{\sigma_A^2}{2}} A_t, \qquad q_t^L = \frac{\beta}{1-\beta} e^{\frac{\sigma_A^2}{2}} A_t,$$

and the bond portfolio is given by

$$b^S = \frac{\kappa F'(K)k}{\beta^2 e^{\frac{\sigma_A^2}{2}}}, \qquad b^L = -\frac{\kappa F'(K)k}{\beta^2 e^{\frac{\sigma_A^2}{2}}},$$

with $B^S = -b^S$ and $B^L = -b^L$. The value of total bond holdings and long-term bond holdings are given by

$$b_{t+1} = -\frac{1}{1-\beta}\kappa F'(K)kA_t, \qquad \phi_{t+1} = -\frac{1}{1-\beta}\kappa\beta^{-1}F'(K)kA_t$$

The equilibrium debt maturity structure of entrepreneurs therefore consists of a fraction $\beta^{-1} > 1$ of long-term debt and a fraction $1-\beta^{-1} < 0$ of short-term debt. Despite being net borrowers,

[11] The value of ω is given in appendix B.

entrepreneurs have a long position in short-term bonds. The intuition for this result has to do with the nature of the risk-sharing problem that the market is trying to solve. The value of the asset side of leveraged entrepreneurs' balance sheet is entirely state-contingent. Therefore, the market is looking for a bond portfolio whose realized one-period ahead return is also fully state-contingent in order to allow entrepreneurs to shift risk to the liability side of their balance sheet and pass it on to households. Yet long-term bonds have a non-state contingent component. This component corresponds to the first payment on the bond, whose value makes up a fraction $1-\beta^{-1}$ of the total value of the long-term bond. By borrowing long-term $\beta^{-1} > 1$ times their net debt and placing $1-\beta^{-1}$ in short-term bonds, entrepreneurs hold a debt portfolio whose realized one-period ahead return is entirely state-contingent. This particular debt portfolio is the only one that can achieve the socially desirable risk-allocation.

Finally, we observe that in the presence of aggregate risk the insurance provided by long-term debt is not free: there is a positive risk premium or *term premium* $\chi_t \equiv E_t[r^L_{t+1}] - r^S_{t+1}$ on long-term bonds in equilibrium, where $r^L_{t+1} \equiv (1+q^L_{t+1})/q^L_t$ and $r^S_{t+1} \equiv 1/q^S_t$ are the returns on long- and short-term bonds. The term premium is given by

$$\chi_t = \frac{e^{\frac{\sigma_A^2}{2}} - e^{-\frac{\sigma_A^2}{2}}}{A_t} > 0$$

This term premium is naturally an increasing function of the volatility of aggregate shocks σ_A, and it is countercyclical:

$$\frac{\partial \chi_t}{\partial \sigma_A} = \sigma_A \frac{e^{\frac{\sigma_A^2}{2}} + e^{-\frac{\sigma_A^2}{2}}}{A_t} > 0, \qquad \text{and} \qquad \frac{\partial \chi_t}{\partial A_t} = -\frac{e^{\frac{\sigma_A^2}{2}} - e^{-\frac{\sigma_A^2}{2}}}{A_t^2} < 0.$$

It is worth mentioning that the perfect risk-sharing result established under the assumptions of proposition 1 continues to hold in the more general case of CRRA utility and serially correlated TFP, but only up to a first-order of approximation. More precisely, as long as enforcement of long-term contracts is costless, households and entrepreneurs share risk equally up to a first-order, i.e. the policy functions for consumption take the following form

$$\hat{C}_t = \hat{A}_t + \eta^C_t + O(\epsilon^2),$$
$$\hat{c}_t = \hat{A}_t + \eta^c_t + O(\epsilon^2),$$

where $\hat{x}_t = \log(x_t/x)$, $O(\epsilon^2)$ represents terms of second- or higher order, and η^x_t are terms linear in the first-order components of endogenous state variables whose values are known as of $t-1$. In these cases, the long-run (zero-order) maturity choice of agents is efficient, and a constant tax on short- or long-term debt cannot lead to Pareto improvements.

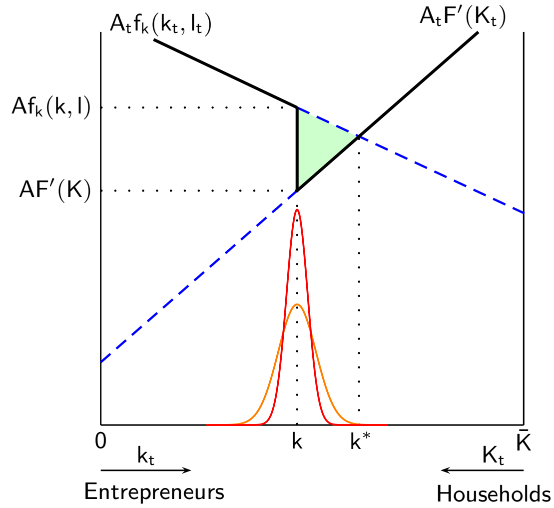

Figure 2: Inefficient capital and risk allocations.

4 Macroprudential policy

This section discusses the role of macroprudential policy in the model, and specifies its objectives and instruments.

4.1 Motivation

In the absence of costly enforcement of long-term debt contracts, the short- and long-term bonds result in approximately effectively complete risk markets.[12] In this case, equilibrium debt portfolio choices result in an allocation of risk that cannot be improved upon using a constant tax on short-term debt. However, when long-term debt is costly to enforce, debt portfolio choices are distorted and a wedge between the private and social benefits of the insurance provided by long-term debt leads to risk allocations that are not constrained efficient. Individual entrepreneurs fail to internalize that by issuing more long-term debt and less short-term debt, they reduce the volatility of the price of capital. This happens because the volatility of the asset price depends on the stability of entrepreneurs' net worth, as the collateral constraint makes the borrowing capacity of entrepreneurs a direct function of their wealth. Relying on long-term rather than short-term debt reduces the exposure of entrepreneurs' net worth to aggregate shocks, and therefore reduces the volatility of the price of capital. A more stable price of capital in turn leads to a more stable distribution of capital between the entrepreneurial and household sectors.

Figure 2 depicts the marginal product of capital in the two sectors. In the deterministic steady-state, capital is allocated inefficiently at k (the efficient capital allocation is at k^*) and the deadweight loss caused by the borrowing constraint corresponds to the area of the shaded triangle.[13] As the deadweight loss is a convex function of the deviation of the capital allocation from the efficient allocation, stable capital allocations are socially more desirable than volatile ones. The

[12]This statement holds exactly only in the case of log utility and serially uncorrelated TFP. It also holds more generally up to a first-order of approximation.

[13]The interpretation is identical to the one in Kiyotaki and Moore (1997).

figure schematically represents two generic ergodic distributions of capital. The distribution with higher variance corresponds to a situation where entrepreneurs rely less on long-term debt and more on short-term debt relative to the distribution with smaller variance. Leveraged borrowers do not generally choose a socially efficient risk exposure because they fail to internalize their contribution to systemic risk. At the margin, relying more on long-term debt and less on short-term debt reduces the volatility of individual net worth. What borrowers fail to internalize, however, is that a lower volatility of individual net worth reduces the volatility of asset prices and thus contributes to lower the volatility of other borrowers' net worth. Entrepreneurs perceive the private insurance benefits of long-term debt issuance, but they fail to recognize its wider social benefits arising from the relevance of the market price of capital for financial constraints. The market failure underlying this inefficiency result is a "fire-sale" externality similar to that emphasized by Lorenzoni (2008) and Korinek (2009a, 2009b). It is a particular application of the general proposition of the constrained suboptimality of competitive equilibria in incomplete markets settings by Stiglitz (1982) and Geanakoplos and Polemarchakis (1986).

4.2 Welfare measures and policy instruments

Let $E_{CE}[u(C) - G(L)]$ and $E_{CE}[u(c)]$ denote the unconditional expected utilities of households and entrepreneurs under the ergodic distribution induced by a competitive equilibrium without government intervention. We consider a government that has the ability to impose a constant tax on entrepreneurs' issuance of short-term debt and to rebate the proceeds of this tax to entrepreneurs.

With macroprudential policy, the entrepreneur's problem is to maximize expected lifetime utility subject to a sequence of collateral constraints and a sequence of budget constraints given by

$$c_t + q_t k_{t+1} + q_t^S b_{t+1}^S + q_t^L b_{t+1}^L = A_t f(k_{t+1}, l_t) - w_t l_t + q_t k_t + (1 + \tau^S) b_t^S + (1 + q_t^L) b_t^L + T_t^E,$$

where τ^S is tax on short-term debt, and T_t^E is a transfer. The household's problem is unaffected. A competitive equilibrium with macroprudential policy is defined by sequences of state-contingent prices $\{w_t, q_t^S, q_t^L, q_t\}_{t=0}^{\infty}$, allocations $\{c_t, C_t, l_t, L_t, b_{t+1}, B_{t+1}, \Phi_{t+1}, \phi_{t+1}, k_{t+1}, K_{t+1}\}_{t=0}^{\infty}$, and policy instruments $(\tau^S, \{T_t^E\}_{t=0}^{\infty})$ such that: (a) households maximize expected lifetime utility subject to their sequence of budget constraints, taking as given prices, policies and initial conditions (B_0, Φ_0, K_0), (b) entrepreneurs maximize expected lifetime utility subject to their sequence of budget and collateral constraints, taking as given prices, policies and initial conditions (b_0, ϕ_0, k_0), (c) the markets for short-term bonds, long-term bonds and capital clear, and (d) the government runs a balanced budget:

$$T_t^E + \tau^S b_t^S = 0.$$

Let $E[u(C) - G(L)]$ and $E[u(c)]$ denote the unconditional expected utilities of households and entrepreneurs under the ergodic distribution induced by a competitive equilibrium with macroprudential policy. We assume that the government's objective in setting macroprudential policy is to maximize the unconditional expected utility of entrepreneurs subject to providing households with an unconditional expected utility at least as high as in a competitive equilibrium without government intervention, i.e. the government solves

$$\max_{\tau^S} E[u(c)] \qquad \text{s.t} \quad E[u(C) - G(L)] \geq E_{CE}[u(C) - G(L)].$$

5 Quantitative analysis

5.1 Solution

Standard perturbation methods for solving dynamic stochastic general equilibrium (DSGE) models are inappropriate to solve the model presented in this paper because of the presence of portfolio choice in an incomplete market setting.[14] Progress has recently been made in this area with the methods proposed by Devereux and Sutherland (2009, forthcoming), Tille and van Wincoop (2008) and Evans and Hnatkovska (2008) to produce approximate solutions for two-country DSGE models featuring portfolio choice under incomplete markets. Despite a collateral constraint and differences in discount factors, the structure of our two-agent model is remarkably similar to the two-country DSGE models for which these methods are designed. We thus use the approach of Devereux and Sutherland (2009, forthcoming) together with the "standard" algorithm of Schmitt-Grohe and Uribe (2004) to obtain a second-order accurate solution of our model. The general principle underlying Devereux and Sutherland's approach is due to Samuelson (1970) and states that in order to derive the solution for portfolio choice up to N-th order accuracy, the portfolio problem must be approximated up to the $N+2$-th order. Appendix C provides details on the model solution.

5.2 Functional forms and calibration

We adopt the following functional forms for utility and producton functions. Utility from consumption takes the standard CRRA form

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma},$$

disutility from labor is assumed to be given by

$$G(L) = \frac{L^{\zeta}}{\zeta},$$

and entrepreneurial production is assumed to be Cobb-Douglas

$$f(k,l) = k^{\alpha_e} l^{1-\alpha_e},$$

[14] The failure of standard perturbation methods in models with portfolio choice is easily understood. These methods usually approximate the model solution around the deterministic steady state. Portfolio choices, however, are not uniquely defined in the deterministic steady state, as assets (i.e. in our case short-term and long-term bonds) are perfect substitutes. Hence, the deterministic steady state does not deliver a natural approximation point. Further, a linearized solution features certainty equivalence, while portfolio choices explicitly depend on the risk characteristics of the available assets.

		Source/target
Steady-state productivity	$A = 1$	Normalization
Fixed capital stock	$\bar{K} = 1$	Normalization
Household's discount factor	$\beta = 0.99$	Standard DSGE value
Entrepreneur's discount factor	$\gamma = 0.98$	Internal rate of return 2x real rate
Coefficient of relative risk aversion	$\sigma = 2$	Standard DSGE value
Elasticity parameter of labor supply	$\zeta = 1.01$	Virtually flat labor supply
Share of collateralizable assets	$\kappa = 0.3$	Debt ratio = 30%, Welch (2004)
Capital share in entrepreneur's output	$\alpha_e = 0.36$	Standard DSGE value
Capital share in home production	$\alpha_h = 0.13$	Greenwood and Hercowitz (1991)
Shift parameter in home production	$\nu = 0.41$	Corporate share of productive capital = 75%
Monitoring cost on LT contracts	$\vartheta = 0.030$	Long-run debt duration = 3.9 years
TFP process	$\rho = 0.57$	Autocorrelation $\rho_Y = 0.84$ and
	$\sigma_A = 0.012$	standard deviation $\sigma_Y = 0.017$ of U.S. GDP

Table 1: Parameter values

and home production is given by

$$F(K) = \nu K^{\alpha_h}.$$

A and $\bar{K}$ are normalized to 1. The parameters β, σ, ζ, α_e are set to standard values from the business cycle literature: $\beta = 0.99$, $\sigma = 2$, $\zeta = 1.01$, $\alpha_e = 0.36$. Following Iacoviello (2005), we set the entrepreneur's discount factor γ to 0.98, which implies an entrepreneurial internal rate of return twice as big as the equilibrium real interest rate. We set κ to 0.3, which matches an entrepreneurial debt ratio (debt over assets) of 30%. Welch (2004) finds a mean debt ratio of 29.8% in a sample of over 60,000 large publicly traded firms in the period 1964-2000. The capital share in home produciton is set to $\alpha_h = 0.13$, as in Greenwood and Hercowitz (1991). The scale factor ν in home production is set to yield a steady state ratio of productive assets held by the corporate sector $k/(K + k)$ of 75%, in line with Flow of Funds data. The monitoring cost parameter θ is set 0.030, implying a long-run average maturity structure (duration) of 3.9 years (corresponding a maturity structure with weights of 0.85 on short-term debt and 0.15 on long-term debt). This seems consistent with the descriptive results in Barclay and Smith (1995) 's study of a sample of a large number of non-financial corporation from 1974 to 1992.[15] Finally, we use simulations to set the parameters governing the TFP process, ρ and σ_A, at values that make the model match the cyclical time series properties of quartely U.S. real GDP for the period 1947-2007. In the data, we find an autocorrelation of GDP of 0.84 and standard deviation of 0.017, and setting the parameters to $\rho = 0.57$ and $\sigma_A = 0.012$ replicates these moments. Table 1 summarizes the calibration.

[15] Standard data sources like COMPUSTAT only report the the percentage of debt that matures in more than one, two, three, four and five years. Uncovering a debt duration from this data is difficult, but a value of 3.9 years seems broadly consistent with the data. Given the uncertainty surrounding this parameter value, it will be worth making robustness checks with respect to it.

5.3 Results

5.3.1 Positive implications of debt maturity

Figure 1 illustrates the impulse responses of some of the model's variables to a 1% negative innovation to log TFP in the benchmark model with only short-term debt, as well as in the multiple maturities model under two different assumptions regarding the enforcement cost of long-term contracts. The responses in the benchmark case with only short-term debt are represented with dashed lines, while those in the model with multiple maturities are represented with solid lines. The light (red) line is the multiple maturities model with costless enforcement of long-term debt ($\vartheta = 0$) and the dark (black) line is the multipled maturities model with costly enforcement with the monitoring cost calibrated to yield an equilibrium maturity structure with a duration of 3.9 years. Several comments are in order. First, we observe that when enforcement of long-term contracts is costless, risk markets are approximately effectively complete and shocks do not cause first-order changes in the wealth distribution. Entrepreneurs and households share risk equally.[16] In this case, capital does not get reallocated much following the shock. Consequently, the path of aggregate output follows mainly the mean-reverting path of TFP. In contrast, when long-term contracts are costly to enforce, borrowers choose more short-term debt in their portfolios, and entrepreneurs suffer relatively more than households from a negative aggregate shock. This disproportionately large deterioration in their net worth tightens their collateral constraint and induces a reallocation of capital towards the unconstrained household sector: fire-sales occur. The standard financial ampliciation mechanism is at work here. The negative shock reduces the entrepreneurs' wealth, thereby reducing their ability to borrow. Facing tightening financial conditions, entrepreneurs fire-sell assets, thus causing further declines in the asset price (as households value the physical capital at a decreasing rate). This decline in the asset price hurts the entrepreneurs' wealth further, leading to yet another round of fire-sales. After the shock and the fire-sales, entrepreneurs slowly rebuild their capital holdings because it takes time for them to re-accumulate net worth and re-establish a borrowing capacity at its pre-shock level. When long-term debt is costly to enforce, the high risk exposure of leveraged borrowers is a source of amplification and persistence of fundamental shocks, as in the benchmark case with only short-term debt.

5.3.2 Normative results

In order to analyze the normative implications of macroprudential policy, we compute welfare by calculating the unconditional expected utility of the households and entrepreneurs using a second-order accurate solution for consumption, and a second-order Taylor expansion of the period utility function. Hence, our welfare measures correctly reflect changes in both the unconditional mean and the unconditional variance of consumption and labor supply brought about by taxes on short-term debt.

[16] This can be observed from the fact that their consumption responses are identical. Again, this holds up to a first-order of approximation only.

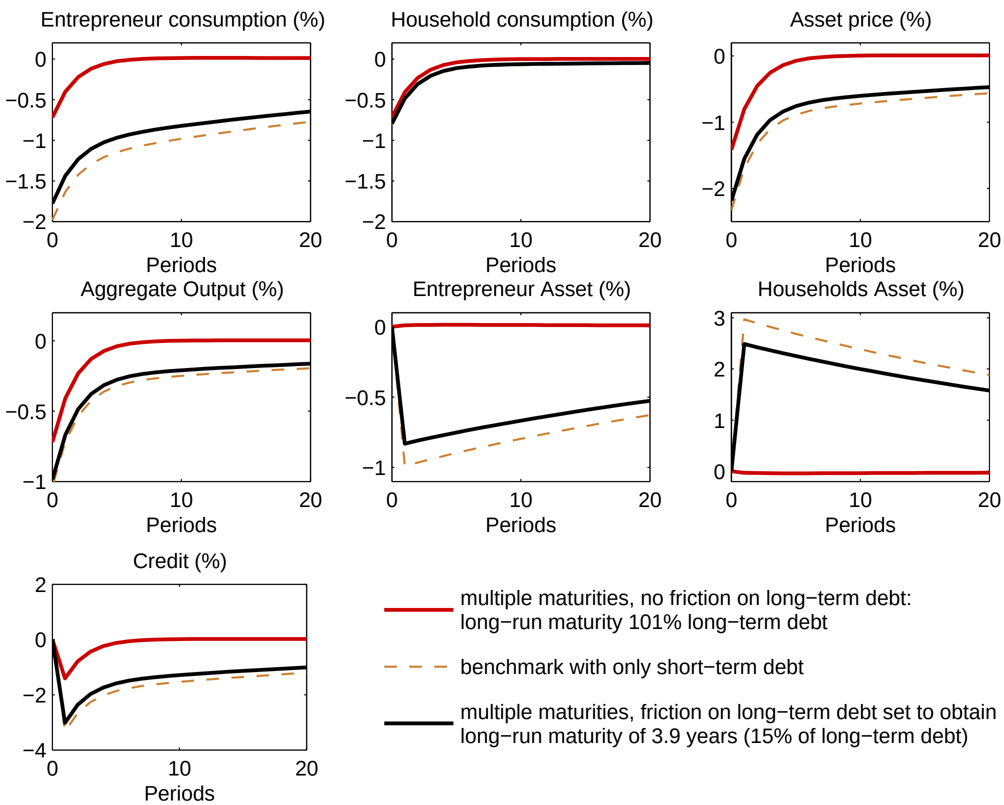

Figure 3: Impulse responses to 1% negative TFP shock in benchmark with only short-term debt, and with multiple maturities (with and without monitoring cost on long-term debt).

In the competitive equilibrium without macroprudential policy, monitoring costs are such that the long-run debt maturity structure consists of 85% of short-term debt and 15% of long-term debt, resulting in a duration of the debt portfolio of 3.9 years. A 100% short-term maturity structure would yield a duration of 1 quarter, while a 100% long-term maturity structure would correspond to a duration of 25 years (i.e. $1/(1-\beta) = 100$ quarters). By taxing entrepreneurs' issuance of short-term debt, a macroprudential authority can effectively lengthen the equilibrium long-run debt maturity structure. Figure 4 shows households' (upper panels) and entrepreneurs' (lower panels) welfare in equivalent consumption terms at different long-run equilibrium maturity structures induced by various levels of taxes on short-term debt. The figure illustrates that by using taxes to lenghten the maturity structure, the macroprudential authority does more than just shifting risk from entrepreneurs to households. Maturity structures longer than in the competitive equilibrium are found to effectively reduce risk and increase welfare for both groups of agents. This result is explained by the positive spill-overs that arise when an individual entrepreneur lenghtens his debt maturity. Such a lengthening leads to a more stable net-worth, and therefore to less fire-sales and to more stable capital holdings at his individual level. This in turn leads to less volatile asset prices, which helps out other entrepreneurs, as well as to more stable wages, which benefits households. Those positive spill-over effects are not perceived by atomistic entrepreneurs in a decentralized equilibrium. In the language of financial markets, each individual entrepreneur fails to internalize the reduction in *systemic risk* that a lengthening of his own maturity structure would bring about.

On the other hand, figure 4 illustrates that the socially desirable maturity structure does not simply coincide with the maturity structure that would arise in the absence of costly enforcement of long-term contracts - a structure consisting of 101% of long-term debt. When long-term debt enforcement is costly, a longer maturity structure is beneficial for risk-sharing purposes, but it entails more resources spent on monitoring expenses. When the macroprudential authority uses taxes to induce agents to substitute short-term debt with long-term debt, it also causes more resources to be wasted in unproductive monitoring costs. The model hence captures what is widely seen as the trade-off faced by policy makers when trying to distort maturity choices away from their competitive equilibrium levels.

The optimal maturity structure in the model consists of 50% of short-term debt and 50% of long-term debt, or a debt duration of 12.5 years. This contrasts remarkably with the debt maturity structure of the competitive equilibrium without government intervention. The optimal constant tax is given by $\tau^S = 0.00013$, which represents an annual tax of 0.052% of the face value of short-term bonds. The optimal tax corresponds to about a half of the term premium implied by the model. The fact that this tax is quantitatively small relates to the equity premium puzzle or its bond counterpart, the term premium puzzle: with CRRA utility, a realistic calibration of the aggregate shock process results in welfare costs of uncertainty that are so low that model-based risk premia are unrealistically small. Accordingly, a tiny tax on short-term debt suffices to move the equilibrium maturity structure by considerable amounts.

To illustrate the implications of macroprudential policy on allocations and prices, figure 5 plots

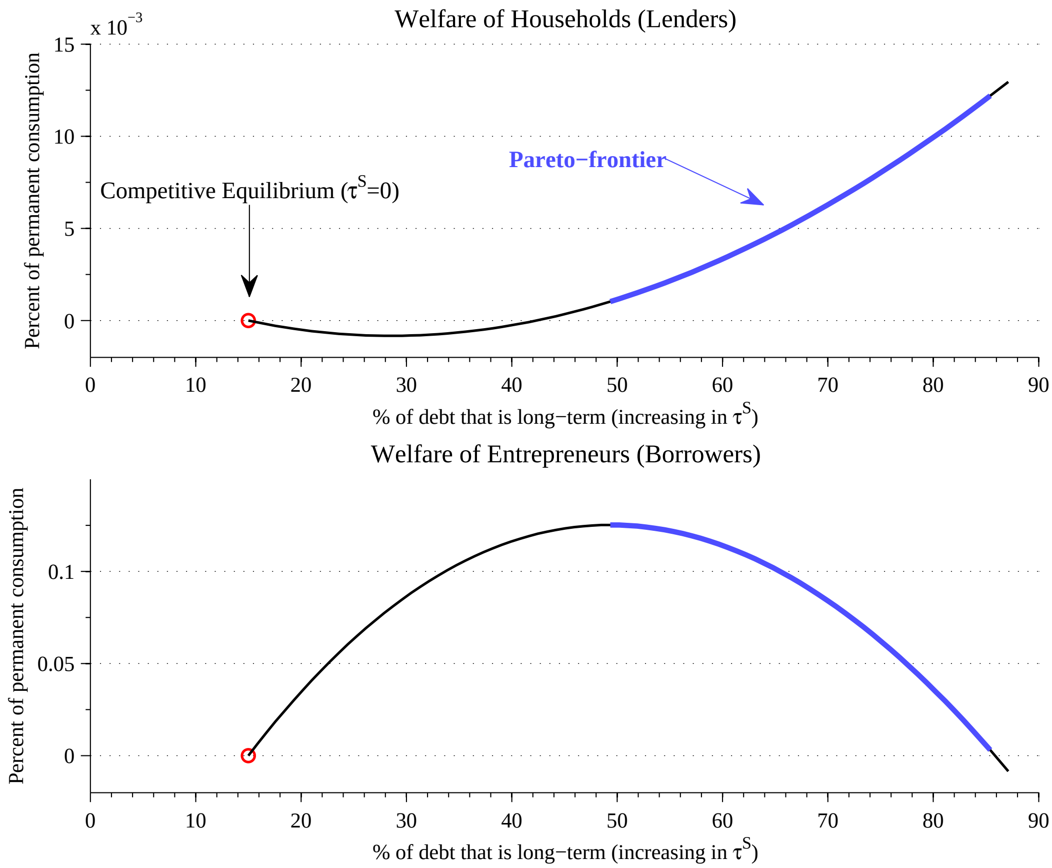

Figure 4: Welfare and debt maturity structure: tax on short-term debt.

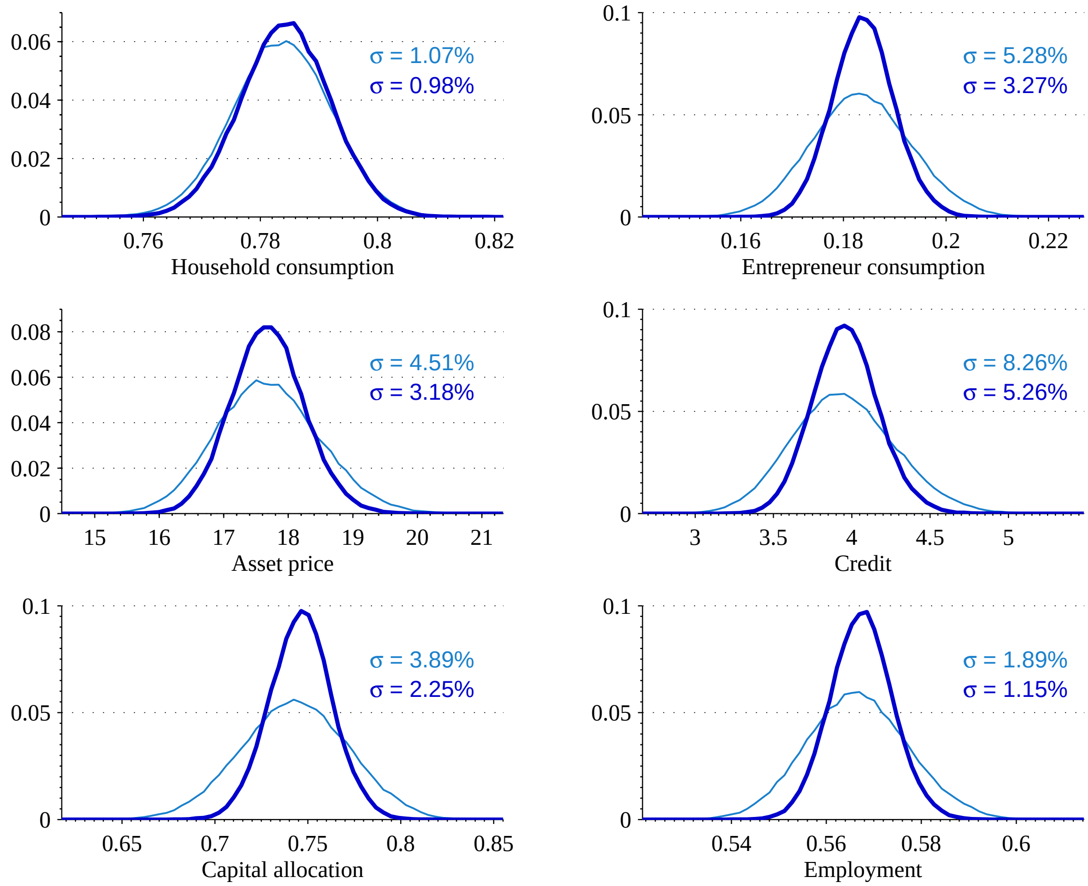

Figure 5: Ergodic densities of the model's variables in competitive equilibrium without macroprudential policy and in competitive equilibrium with macroprudential policy (socially optimal tax on short-term debt).

ergodic densities for some of the variables based on simulations for 100,000 periods of the model without government intervention and with the tax on short-term debt set at its optimal level. The first and second numbers on the right of the densities are the variables' standard deviations in the cases without government intervention and with the optimal tax on short-term debt, respectively. The figure reveals that the tax on short-term debt significantly reduces the volatility of entrepreneurial net-worth (and therefore consumption), asset prices, credit, employment and of the capital allocation across the two sectors, while it only marginally reduces consumption risk for households.

The consequences of a tax on short-term debt is further illustrated in figure 6, where the ergodic densities of output with and without the tax on short-term debt are displayed. The standard deviation of output is reduced by the tax from 1.73% to 1.31% - a 25% decrease. Moreover, when a crisis is defined as an output level of less than one standard deviation below mean output in the laissez-faire case, we find that the tax on short-term debt divides the long-run probability of a crisis by a factor of eight, lowering it from 2.33% to 0.28%. The model hence predicts that taxing

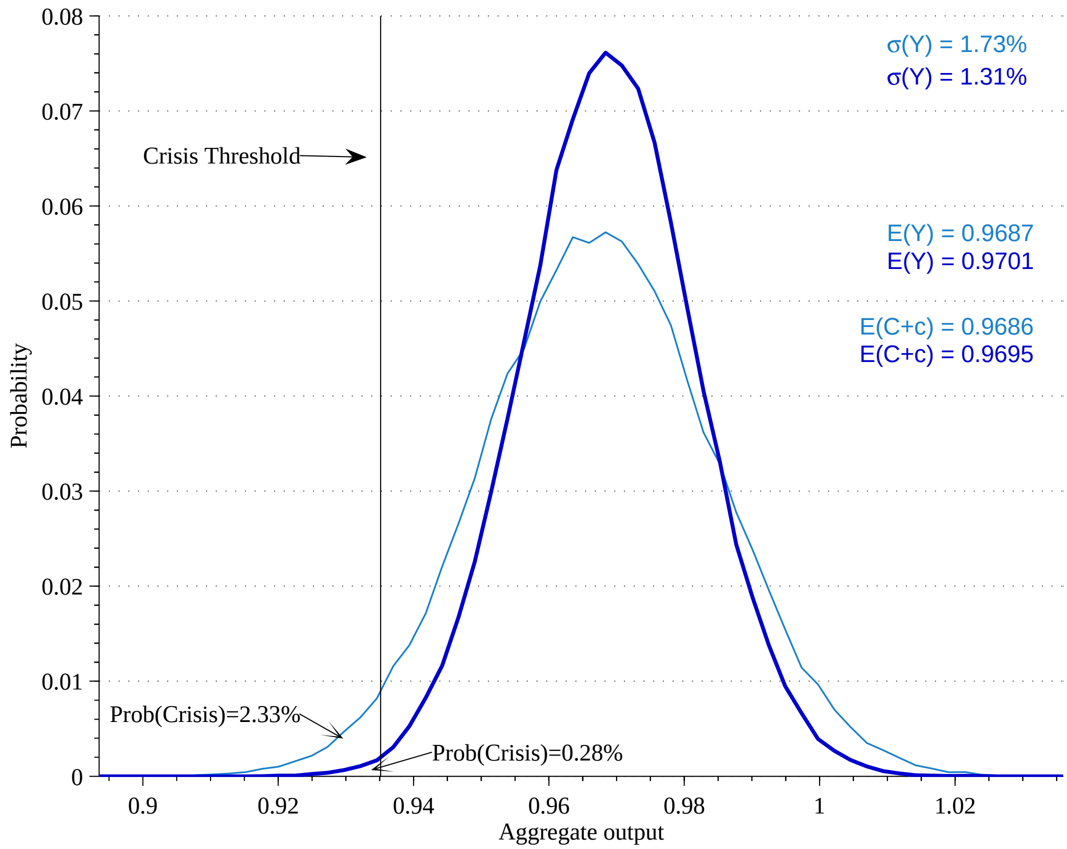

Figure 6: Ergodic density of Aggregate Output in competitive equilibrium without macroprudential policy and in competitive equilibrium with macroprudential policy (socially optimal tax on short-term debt).

short-term debt can bring about a quantitatively significant reduction in the incidence of crises.

As mentioned in section 4.1, less volatile factor allocations also contribute to reduce the dead-weight loss caused by the collateral constraint. This effect can be seen from the 0.14% increase in long-run mean output brought about by the tax on short-term debt. Interestingly, even when the additional monitoring costs caused by a lengthening of the maturity structure are subtracted from aggregate income, the long-run mean net-output (available for consumption since there is no investment) is still 0.09% higher with the optimal tax on short-term debt than in the laissez-faire case. In the framework of the model, the reduction in macroeconomic volatility achieved by taxing short-term debt and inducing agents to rely on longer term financing does not come at the expense of lower mean output, even after netting out the increased resource loss associated with more monitoring activities.

6 Conclusion

This paper develops a quantitative general equilibrium model of the determinants of private agents' debt maturity. By featuring a number of practically relevant financial frictions, the model captures the largely agreed on trade-off between the insurance benefits of long-term debt and the cost benefits of short-term debt. The analysis indicates that when asset prices affect financing constraints in the economy, debt maturity choices made in decentralized markets are generally not constrained efficient. In particular, the results suggest that individual borrowers issue too much short-term debt and too little long-term debt in a competitive equilibrium, as they fail to internalize the reduction in systemic risk brought about by a lengthening of their debt maturity structure. The paper also shows that a constant tax on short-term debt can lead to Pareto improvements and result in substantially less volatile allocations and asset prices. The analysis hence provides a theoretical foundation for a macroprudential policy framework that would penalize short-term debt.

References

Aiyagari, S. R. and M. Gertler (1999). Overreaction of asset prices in general equilibrium. *Review of Economic Dynamics 2*, 3–35.

Angeletos, G.-M. (2002). Fiscal policy with noncontingent debt and the optimal maturity structure. *Quarterly Journal of Economics 117*(3), 1105–1131.

Barclay, M. and C. Smith (1995). The maturity structure of corporate debt. *Journal of Finance L*, 609–631.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctualtions. *American Economic Review 79*, 1–15.

Bernanke, B., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantatative business cycle framework. In J. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1, pp. 1341–1393. Elsevier Science B.V.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Broner, F., G. Lorenzoni, and S. Schmukler (2008). Why do emerging economies borrow short term. Mimeo.

Brunnermeier, M., A. Crocket, C. Goodhart, A. D. Persaud, and H. Shin (2009). The fundamental principles of financial regulation. Icmb-cepr geneva report on the world economy 11.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives 23*(1), 77–100.

Buera, F. and J. P. Nicolini (2004). Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics 51*, 531–554.

Calomiris, C. and C. Kahn (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review 81*(3), 497–513.

Carlstrom, C. T. and T. S. Fuerst (1997). Agency costs, net worth, and business fluctualtions: A computable general equilibrium analysis. *American Economic Review 87*, 893–910.

Devereux, M. and A. Sutherland (2010). Country portfolio dynamics. *Journal of Economic Dynamics and Control 34*(7), 1325–1342.

Devereux, M. and A. Sutherland (2011). Country portfolios in open economy macro models. *Journal of the European Economic Association 9*, 337–389.

Diamond, D. (1991). Debt maturity structure and liquidity risk. *Quarterly Journal of Economics 106*(3), 709–737.

Diamond, D. and P. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Diamond, D. and R. Rajan (2001). Liquidity risk, liquidity creation and financial fragility: A theory of banking. *Journal of Political Economy 109*(2), 287–327.

Evans, M. D. D. and V. Hnatkovska (2008). A method for solving general equilibrium models with incomplete markets and many financial assets. Mimeo, UBC.

Flannery, M. (1986). Asymmetric information and risky debt maturity choice. *Journal of Finance 41*(1), 19–37.

Geanakoplos, J. and H. Polemarchakis (1986). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In W. A. Barnett and K. J. Singleton (Eds.), *Uncertainty, Information and Communication, Essays in Honor of Kenneth J. Arrow*. Cambridge University Press.

Greenwood, J. and Z. Hercowitz (1991). The allocation of capital and time over the business cycle. *The Journal of Political Economy 99*, 1188–1214.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review 95*, 739–764.

Jeanne, O. (2009). Debt maturity and the international financial architecture. *American Economic Review 99*(5), 2135–2148.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*, 211–248.

Kiyotaki, N. and J. Moore (2003). Inside money and liquidity. Mimeo, London School of Economics.

Kiyotaki, N. and J. Moore (2005). Financial deepening. *Journal of the European Economic Association 3*(2–3), 701–713.

Korinek, A. (2009). Excessive dollar borrowing in emerging markets: Balance sheet effects and macroeconomic externalities. University of maryland working paper.

Korinek, A. (2011). Systemic risk-taking: Amplification effects, externalities, and regulatory responses. Mimeo, University of Maryland.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies 75*(3), 809–833.

Mendoza, E. G. and K. A. Smith (2006). Quantitative implications of a debt-deflation theory of sudden stops and asset prices. *Journal of International Economics 70*, 82–114.

Rodrick, D. and A. Velasco (1999). Short-term capital flows. In B. Pleskovic and J. E. Stiglitz (Eds.), *Annual World Bank Conference on Development Economics*. The World Bank.

Samuelson, P. (1970). The fundamental approximation theorem of portfolio analysis in terms of means, variance and higher moments. *Review of Economic Studies 37*, 537–542.

Schmitt-Grohe, S. and M. Uribe (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*, 755–775.

Stiglitz, J. (1982). The inefficiency of stock market equilibrium. *Review of Economic Studies 49*, 241–261.

Tille, C. and E. van Wincoop (2008). International capital flows. Mimeo, University of Virginia.

Tirole, J. (2003). Inefficient foreign borrowing: A dual- and common-agency perspective. *American Economic Review 93*(5), 1678–1702.

Welch, I. (2004). Capital structure and stock returns. *The Journal of Political Economy 112*, 106–131.

A Deterministic steady-state

This appendix gives details on the model's deterministic steady state.

Combining entrepreneur's optimal labor demand condition with household's optimal labor supply condition, we get

$$Af_l(k,l) = \frac{G'(l)}{u'(C)}. \tag{A.12}$$

From the households' first-order conditions, the steady-state asset prices are given by $q^S = \beta$, $q^L = \beta/(1-\beta)$ and $q = AF'(K)/(1-\beta)$. From the entrepreneurs' first-order conditions, the steady-state multiplier is given by $\mu = (1-\gamma/\beta)u'(c)$ and the asset prices must satisfy

$$q = Af_k(k,l)/[1-\gamma-\kappa(1-\gamma/\beta)].$$

Combining the two expressions for the asset price q, we obtain

$$AF'(K) = \frac{\beta(1-\beta)}{\beta(1-\beta)+(\beta-\kappa)(\beta-\gamma)} Af_k(k,l)$$

Now, given that the entrepreneur's collateral constraint binds, the market value of debt is given by

$$\beta B^S + \frac{\beta}{1-\beta} B^L = -\beta b^S - \frac{\beta}{1-\beta} b^L = \kappa q k$$

and therefore

$$B^S + \frac{1}{1-\beta} B^L = -b^S - \frac{1}{1-\beta} b^L = \beta^{-1}\kappa q k.$$

From the household's budget constraint, steady state household consumption C is given by

$$C = AF(K) + Af_l(k,l)l + \beta^{-1}\kappa AF'(K)k.$$

Hence, the steady state allocation of physical capital and labor supply are a solution to the two equations

$$F'(K) = \frac{\beta(1-\beta)}{\beta(1-\beta)+(\beta-\kappa)(\beta-\gamma)} f_k(k,l)$$

and

$$Af_l(k,l) = \frac{G'(l)}{u'\left(AF(K) + Af_l(k,l)l + \beta^{-1}\kappa AF'(K)k\right)}.$$

For further references, it is useful to define the household's beginning of period wealth as $Z_t \equiv q_t K_t + B_t^S + (1+q_t^L)B_t^L$ and the entrepreneur's beginning of period wealth as $z_t \equiv q_t k_t + b_t^S + (1+q_t^L)b_t^L$. Also, define aggregate wealth as $\Omega_t \equiv Z_t + z_t$. The steady state wealth positions are given by

$$\begin{aligned} Z &= qK + \beta^{-1}\kappa q k \\ z &= qk - \beta^{-1}\kappa q k. \end{aligned}$$

Hence, the relative wealth positions are given by

$$\omega = \frac{z}{Z+z} = \frac{(1-\beta^{-1}\kappa)k}{K+k}$$

It is also useful to define the entrepreneur's share of consumption in aggregate output as $\tilde{\omega} \equiv c_t/[Y_t A_t]$ and the according household's share of consumption in aggregate output as $(1-\tilde{\omega}) \equiv C_t/[Y_t A_t]$. In the steady state, these shares are given by

$$\tilde{\omega} = \frac{f(k,l) - f_l(k,l)l - \beta^{-1}\kappa F'(K)k}{F(K) + f(k,l)}$$

and

$$1-\tilde{\omega} = \frac{F(K) + f_l(k,l)l + \beta^{-1}\kappa F'(K)k}{F(K) + f(k,l)}.$$

B Proofs

B.1 Proof of Proposition 1

We show here that under the assumptions of log utility and serially uncorrelated TFP, the costless enforcement model results in effectively complete risk markets, perfect risk sharing between entrepreneurs and households, and no persistence in allocations and asset prices. We also show that the entrepreneur's collateral constraint binds in every period and state. We proceed by first postulating allocations and prices, and then showing that given these prices, the allocations satisfy both agents' optimality conditions and constraints.

Assume $u(\cdot) = \ln(\cdot)$, $\rho = 0$ and $\vartheta = 0$. Consider initial values for capital and bond allocations that result in a distribution of wealth corresponding to the one prevailing in the deterministic steady state (see Appendix A), i.e. $Z_0 = (1-\omega)\Omega_0$ and $z_0 = \omega\Omega_0$, where $\Omega_0 \equiv Z_0 + z_0$.

Define $Y_t \equiv F(K_t) + f(k_t, l_t)$, denote by k and K the allocation of capital in the deterministic steady state and by l employment in the deterministic steady-state. We conjecture the following allocations: $k_t = k$, $K_t = K$, $l_t = 1$, $c_t = \tilde{\omega} Y A_t$ and $C_t = (1-\tilde{\omega})YA_t$ (see Appendix A for the value of $\tilde{\omega}$). It is straightforward to verify that with those allocations, both agents' optimality conditions for labor supply/demand, Euler equations for capital, short-term bonds and long-term bonds are satisfied when asset prices are given by

$$q_t = \frac{1}{1-\beta}F'(K)A_t, \qquad q_t^S = \beta A_t E_t[e^{-\epsilon_{t+1}^A}], \qquad q_t^L = \frac{\beta}{1-\beta}A_t E_t[e^{-\epsilon_{t+1}^A}],$$

and the wage is given by

$$w_t = A_t f_l(k,l),$$

provided that the entrepreneurs' collateral constraint binds as the multiplier on the constraint satisfies

$$\mu_t = (1-\gamma/\beta)/(\omega Y A_t).$$

Also, conjecture that bond holdings are time invariant and given by

$$b^S = \frac{\kappa F'(K)k}{\beta^2 E_t[e^{-\epsilon_{t+1}^A}]}, \qquad b^L = -\frac{\kappa F'(K)k}{\beta^2 E_t[e^{-\epsilon_{t+1}^A}]},$$

and $B_S = -b_S$, $B_L = -b_L$.

It is straightforward to verify that the two agents budget constraints and the entrepreneurs' borrowing constraint hold with equality in every period and state. By showing that both agents' optimality conditions, budget constraints and borrowing constraints all hold with equality at the candidate allocations and prices, we have established that these constitute a competitive equilibrium.

The allocations are identical to the ones obtained in an environment with a full set of Arrow securities, where the household's and entrepreneur's budget constraints would be given by

$$\begin{aligned} C(s^t) + q(s^t)K(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)B(s_{t+1}, s^t) &= A(s^t)F(K(s^t)) + w(s^t)L(s^t) + q(s^t)K(s^{t-1}) + B(s^t) \\ c(s^t) + q(s^t)k(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}, s^t) &= A(s^t)f(k(s^t), l(s^t)) - w(s^t)l(s^t) + q(s^t)k(s^{t-1}) + b(s^t), \end{aligned}$$

and the entrepreneur's borrowing constraint would be

$$-\sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}, s^t) \leq \kappa q(s^t)k(s^t).$$

In these expressions, s_t has the usual interpretation of the state at time t, while s^t describes the entire history up to time t. $B(s_{t+1}, s^t)$ and $b(s_{t+1}, s^t)$ denote the holdings of recursive Arrow securities available for trade at time t, and $q(s_{t+1}, s^t)$ denotes the price of these securities.

C Model solution under incomplete markets

This appendix gives some details on the solution of the model when markets are incomplete. The solution approach follows the one developed by Devereux and Sutherland (2010) and Devereux and Sutherland (2011) and is adapted to be used together with the standard algorithm of Schmitt-Grohe and Uribe (2004) that computes first- and second-order approximations to the policy function in DSGE models without portfolio choices.

For future reference, we distinguish between portfolio variables and non-portfolio variables. The welfare calculations of Section 5 require a second-order accurate solution for households' consumption C_t and entrepreneurs' consumption c_t, which are both non-portfolio variables. As the model presented in this paper belongs to a general class of models in which second-order accurate solutions for non-portfolio variables only require a first-order accurate solution for portfolio variables, we only need to solve for the debt portfolio up to a first-order. Following Devereux and Sutherland (2009, forthcoming), we do this in steps. We first solve for the zero-order of the debt portfolio, and then use this zero-order portfolio to obtain a first-order accurate solution for non-portfolio variables. We then use this first-order solution to solve for the first-order portfolio. Finally, we use the first-order portfolio to obtain a second-order accurate solution for non-portfolio variables.

C.1 Zero-order portfolio and first-order non-portfolio variables

Solving for ϕ

Rewrite the model equilibrium conditions as

$$
\begin{aligned}
ce^{\hat{c}_t} + qe^{\hat{q}_t} ke^{\hat{k}_{t+1}} + (\hat{b}_{t+1} + b) &= \alpha_e A e^{\hat{A}_t} k^{\alpha_e} e^{\alpha_e \hat{k}_{t+1}} l^{1-\alpha_e} e^{(1-\alpha_e)\hat{l}_t} + qe^{\hat{q}_t} ke^{\hat{k}_t} \\
&\quad + r^S e^{\hat{r}_{St}} (\hat{b}_t + b) + \tau^S r^S e^{\hat{r}_t^S} (\hat{b}_t + b - \hat{\phi}_t - \phi) \\
&\quad + \beta^{-1} (e^{\hat{r}_{Lt}} - e^{\hat{r}_{St}})(\hat{\phi}_t + \phi) \\
Ae^{\hat{A}_t} k^{\alpha_e} e^{\alpha_e \hat{k}_{t+1}} l^{1-\alpha_e} e^{(1-\alpha_e)\hat{l}_t} + A\nu e^{\hat{A}_t} K^{\alpha_h} e^{\alpha_h \hat{k}_{t+1}} &= ce^{\hat{c}_t} + Ce^{\hat{C}_t} - \left(\epsilon_t^{\vartheta}\right)^2 \frac{1}{q^L e^{\hat{q}_{t-1}^L}} (\hat{\phi}_t + \phi) \\
(\hat{b}_{t+1} + b) + (\hat{B}_{t+1} + B) &= 0 \\
ke^{\hat{k}_{t+1}} + Ke^{\hat{K}_{t+1}} &= \bar{K} \\
-(\hat{b}_{t+1} + b) &= \kappa q e^{\hat{q}_t} ke^{\hat{k}_{t+1}} \\
\frac{qe^{\hat{q}_t} - Ae^{\hat{A}_t} \nu \alpha_h K^{\alpha_h - 1} e^{(\alpha_h - 1)\hat{K}_{t+1}}}{C^\sigma e^{\sigma \hat{C}_t}} &= \beta E_t \left[\frac{qe^{\hat{q}_{t+1}}}{C^\sigma e^{\sigma \hat{C}_{t+1}}} \right] \\
\frac{1}{C^\sigma e^{\sigma \hat{C}_t}} &= \beta r^S e^{\hat{r}_{St+1}} E_t \left[\frac{1}{C^\sigma e^{\sigma \hat{C}_{t+1}}} \right] \\
\frac{1}{C^\sigma e^{\sigma \hat{C}_t}} &= \beta E_t \left[r^L e^{\hat{r}_{Lt+1}} \frac{1}{C^\sigma e^{\sigma \hat{C}_{t+1}}} \right] - \beta E_t \left[\frac{\left(\epsilon_t^{\vartheta}\right)^2}{q^L e^{\hat{q}_t^L}} \frac{1}{C^\sigma e^{\sigma \hat{C}_{t+1}}} \right] \\
\frac{qe^{\hat{q}_t} - Ae^{\hat{A}_t} \alpha_e k^{\alpha_e - 1} e^{(\alpha_e - 1)\hat{k}_{t+1}} l^{1-\alpha_e} e^{(1-\alpha_e)\hat{l}_t}}{c^\sigma e^{\sigma \hat{c}_t}} &= \gamma E_t \left[\frac{qe^{\hat{q}_{t+1}}}{c^\sigma e^{\sigma \hat{c}_{t+1}}} \right] + \kappa \mu e^{\hat{\mu}_t} \\
\frac{1}{c^\sigma e^{\sigma \hat{c}_t}} &= \gamma (1 + \tau^S) r^S e^{\hat{r}_{St+1}} E_t \left[\frac{1}{c^\sigma e^{\sigma \hat{c}_{t+1}}} \right] + \mu e^{\hat{\mu}_t} \\
\frac{1}{c^\sigma e^{\sigma \hat{c}_t}} &= \gamma E_t \left[r^L e^{\hat{r}_{Lt+1}} \frac{1}{c^\sigma e^{\sigma \hat{c}_{t+1}}} \right] + \mu e^{\hat{\mu}_t} \\
E_t \hat{A}_{t+1} &= \rho \hat{A}_t \\
r^L e^{\hat{r}_t^L} &= \frac{1 + q^L e^{\hat{q}_t^L}}{q^L e^{\hat{q}_{t-1}^L}} \\
Ae^{\hat{A}_t} (1 - \alpha_e) k^{\alpha_e} e^{\alpha_e \hat{k}_{t+1}} l^{-\alpha_e} e^{-\alpha_e \hat{l}_t} &= l^{\zeta - 1} e^{(\zeta - 1)\hat{l}_t} C^\sigma e^{\sigma \hat{C}_t}
\end{aligned}
$$

where $\hat{x}_t \equiv \ln(x_t/x)$ for all variables except ϕ_t and b_t, for which $\hat{\phi}_t \equiv \phi_t - \phi$ and $\hat{b}_t \equiv b_t - b$. Taking first-order Taylor expansions of the above equations results in a linear system where $\hat{\phi}_t$ does not appear and ϕ appears only in the first equation

$$c\hat{c}_t + qk\hat{k}_{t+1} + \hat{b}_{t+1} = \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + r^S b \hat{r}_{St} + r^S \hat{b}_t + \beta^{-1} \phi (\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^2),$$

where $O(\epsilon^2)$ denotes terms second and higher orders. The first-order system cannot be simply solved as in a model without portfolio choice, because the zero-order portfolio ϕ is unknown. However, the first-order system can be solved conditional on some ϕ. From the first-order approximation of the households' two Euler equation for short- and long-term bonds, we notice that $E_t r_{Lt+1} = r_{St+1} + O(\epsilon^2)$. Hence, the excess return on the portfolio $\beta^{-1}\phi(\hat{r}_{Lt} - \hat{r}_{St})$ is a zero mean i.i.d. random variable up to a first-order of approximation. Devereux and Sutherland's approach consists in replacing this term by an exogenous zero mean i.i.d. random variable, solving the first-order approximation of the model, later on recognizing that this term is the excess return and using the first-order solution of the modified model together with a second-order approximation of the agents' Euler equations to solve for the zero-order portfolio. To operationalize this approach using the standard algorithm of Schmidt-Grohe and Uribe (SGU), we alternatively replace the excess return term in the exact entrepreneur's budget constraint by an exogenous variables $\hat{\xi}_t$, so that

$$\hat{\xi}_t \equiv \beta^{-1}(e^{\hat{r}_{Lt}} - e^{\hat{r}_{St}})(\hat{\phi}_t + \phi).$$

A first-order approximation of the entrepreneur's budget constraint then gives

$$c\hat{c}_t + qk\hat{k}_{t+1} + \hat{b}_{t+1} = \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + r^S b\hat{r}_{St} + r^S\hat{b}_t + \hat{\xi}_t + O(\epsilon^2),$$

while a first-order approximation of the definition of $\hat{\xi}_t$ yields

$$\hat{\xi}_t = \beta^{-1}\phi(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^2).$$

We thus replace the excess return term by $\hat{\xi}_t$ in the entrepreneur's budget constraint and append $E_t\hat{\xi}_{t+1} = 0$ to the set of equilibrium conditions of the model. Denote by $\tilde{\epsilon}_t$ the vector of actual exogenous shocks in the model (so far $\tilde{\epsilon}_t = [\epsilon_t^A, \epsilon_t^\vartheta]'$). Also define $\epsilon_t \equiv [\epsilon_{0t}, \tilde{\epsilon}_t']'$. The first-order accurate solution of the modified model using the SGU algorithm is given by

$$\begin{aligned} y_t &= g_x x_t + O(\epsilon^2), \\ x_{t+1} &= h_x x_t + \eta\sigma\epsilon_{t+1} + O(\epsilon^2), \end{aligned}$$

where x_t is a vector of (exogenous and endogenous) state variables, y_t is a vector of control variables and g_x, h_x are coefficient matrices. The solution for the control variables can thus also be written as

$$y_t = g_x h_x x_{t-1} + g_x\eta\sigma\epsilon_t + O(\epsilon^2)$$

Defining $\Theta_1 \equiv g_x h_x$ and $\Theta_2 \equiv g_x\eta\sigma$, the solution for the control variables can be expressed as

$$y_t = \Theta_1 x_{t-1} + \Theta_2\epsilon_t + O(\epsilon^2)$$

In tensor notation and removing time subscripts, we have

$$[y']^i = [\Theta_1]_a^i[x]^a + [\Theta_2]_c^i[\epsilon']^c + O(\epsilon^2)$$

By extracting the appropriate rows of this solution, we can write

$$\begin{aligned} (\sigma\hat{C}' - \sigma\hat{c}') &= [\Theta_1]_a^1[x]^a + [\Theta_2]_c^1[\epsilon']^c + O(\epsilon^2) \\ (\hat{r}_L' - \hat{r}_S') &= [\Theta_2]_c^2[\epsilon']^c + O(\epsilon^2) \end{aligned}$$

We now split ϵ into ϵ_0 and $\tilde{\epsilon}$ and define $[\Theta_2^0]^i \equiv [\Theta_2]_1^i$, $[\tilde{\Theta}_2]_c^i \equiv [\Theta_2]_{c+1}^i$. Hence, we can write the two expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}_L' - \hat{r}_S')$ as

$$\begin{aligned} (\sigma\hat{C}' - \sigma\hat{c}') &= [\Theta_1]_a^1[x]^a + [\Theta_2^0]^1\epsilon_0' + [\tilde{\Theta}_2]_c^1[\tilde{\epsilon}']^c + O(\epsilon^2) \quad &(C.13) \\ (\hat{r}_L' - \hat{r}_S') &= [\Theta_2^0]^2\epsilon_0' + [\tilde{\Theta}_2]_c^2[\tilde{\epsilon}']^c + O(\epsilon^2) \quad &(C.14) \end{aligned}$$

Now we use the fact that $\epsilon'_0 = \hat{\xi}' = \phi\beta^{-1}(\hat{r}'_L - \hat{r}'_S) + O(\epsilon^2)$ in (C.13) and (C.14) to get

$$(\sigma\hat{C}' - \sigma\hat{c}') = [\Theta_1]^1_a[x]^a + \left[\frac{\phi\beta^{-1}[\Theta_2^0]^1[\tilde{\Theta}_2]^2_c}{1-\phi\beta^{-1}[\Theta_2^0]^2} + [\tilde{\Theta}_2]^1_c\right][\tilde{\epsilon}']^c + O(\epsilon^2) \quad \text{(C.15)}$$

$$(\hat{r}'_L - \hat{r}'_S) = \frac{[\tilde{\Theta}_2]^2_c}{1-\phi\beta^{-1}[\Theta_2^0]^2}[\tilde{\epsilon}']^c + O(\epsilon^2) \quad \text{(C.16)}$$

Combining the second-order approximations to the household's and entrepreneur's portfolio equations yields:

$$E\left[\left(\sigma\hat{C}' - \sigma\hat{c}'\right)(\hat{r}'_L - \hat{r}'_S)\right] = -(1-\beta)\sigma^2_\vartheta + \tau^S + O(\epsilon^3)$$

Substituting the first-oder accurate expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ into this portfolio equation leads to the following quadratic equation for the zero-order portfolio

$$m\phi^2 + n\phi + p = 0 + O(\epsilon^3),$$

where

$$\begin{aligned}
m &= [(1-\beta)\sigma^2_\vartheta - \tau^S]\beta^{-2}\left([\Theta_2^0]^2\right)^2 \\
n &= \beta^{-1}\left([\Theta_2^0]^1[\tilde{\Theta}_2]^2_c - [\Theta_2^0]^2[\tilde{\Theta}_2]^1_c\right)[\tilde{\Theta}_2]^2_d[\Sigma]^{cd} - 2[(1-\beta)\sigma^2_\vartheta - \tau^S]\beta^{-1}[\Theta_2^0]^2 \\
p &= [\tilde{\Theta}_2]^1_c[\tilde{\Theta}_2]^2_d[\Sigma]^{cd} + (1-\beta)\sigma^2_\vartheta - \tau^S
\end{aligned}$$

The two solutions are given by

$$\phi = \frac{-n \pm \sqrt{n^2 - 4mp}}{2m} + O(\epsilon)$$

Hence unless $[\Theta_2^0]^2 = 0$ (the excess return does not depend on the portfolio up to a first-order), the monitoring cost potentially introduces multiple equilibrium zero-order portfolios. In our case, only one of the two solutions of the quadratic equation is a valid equilibrium, but it is worth noting that this is not always the case. In the costless enforcement case where $(1-\beta)\sigma^2_\vartheta = 0$ and short-term debt is not taxed ($\tau^S = 0$) the zero-order portfolio is given by the solution of a linear equation:

$$\phi = \beta\frac{[\tilde{\Theta}_2]^1_c[\tilde{\Theta}_2]^2_d[\Sigma]^{cd}}{\left([\Theta_2^0]^2[\tilde{\Theta}_2]^1_c - [\Theta_2^0]^1[\tilde{\Theta}_2]^2_c\right)[\tilde{\Theta}_2]^2_d[\Sigma]^{cd}} + O(\epsilon),$$

which is analogous to the expression in Devereux and Sutherland (2009).

First-order solution for non-portfolio variables, once ϕ is known

Once ϕ has been computed, we can go back to the original model, replace ϕ_t by ϕ in the entrepreneur's budget constraint. Note that this is not a correct equilibrium condition, but just an artifice to solve the model using the SGU algorithm. Since linearizing this artificial equation gives the same expression as a linearization of the true budget constraint, as far as the first-order accuracy is concerned, we can solve the model using this modified budget constraint.

C.2 First-order portfolio and second-order non-portfolio variables

We are now interested in obtaining a first-order accurate solution for ϕ_t and a second-order accurate solution for non-portfolio variables.

First-order solution for ϕ

Taking a second order approximation to the entrepreneur's budget constraint yields

$$\begin{aligned} c\hat{c}_t + \frac{1}{2}c\hat{c}_t^2 + qk\hat{k}_{t+1} + \frac{1}{2}qk\hat{k}_{t+1}^2 + qk\hat{q}_t\hat{k}_{t+1} + \hat{b}_{t+1} \quad = \quad & \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + qk\hat{k}_t \\ & + \frac{1}{2}qk\hat{k}_t^2 + qk\hat{q}_t\hat{k}_t + \tau^S r^S(b-\phi) + \beta^{-1}b\hat{r}_{St} + \frac{1}{2}r^S b\hat{r}_{St}^2 \\ & + r^S\hat{b}_t + r^S\hat{r}_{St}\hat{b}_t + \phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) \\ & + \frac{1}{2}\phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2) + \beta^{-1}\hat{\phi}_t(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^3) \end{aligned}$$

We now postulate that $\hat{\phi}_t$ is linear in the model's state variables:

$$\hat{\phi}_t = [\psi]_k[x_{t-1}]^k + O(\epsilon^2)$$

Following Devereux and Sutherland, we again replace the excess return term in the original entrepreneur's budget constraint by

$$\hat{\xi}_t \equiv \beta^{-1}(e^{\hat{r}_{Lt}} - e^{\hat{r}_{St}})(\hat{\phi}_t + \phi).$$

A second-order approximation of the modified entrepreneur's budget constraint gives

$$\begin{aligned} c\hat{c}_t + \frac{1}{2}c\hat{c}_t^2 + qk\hat{k}_{t+1} + \frac{1}{2}qk\hat{k}_{t+1}^2 + qk\hat{q}_t\hat{k}_{t+1} + \hat{b}_{t+1} \quad = \quad & \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + qk\hat{k}_t \\ & + \frac{1}{2}qk\hat{k}_t^2 + qk\hat{q}_t\hat{k}_t + \tau^S r^S(b-\phi) + r^S b\hat{r}_{St} + \frac{1}{2}r^S b\hat{r}_{St}^2 \\ & + r^S\hat{b}_t + r^S\hat{r}_{St}\hat{b}_t + \hat{\xi}_t + O(\epsilon^3) \end{aligned}$$

while a second-order approximation of the definition of $\hat{\xi}_t$ yields

$$\hat{\xi}_t = \phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) + \phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2) + \beta^{-1}\hat{\phi}_t(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^3).$$

Now, given that $\hat{\phi}_t$ is a function of x_{t-1}, the term $\beta^{-1}\hat{\phi}_t(\hat{r}_{Lt} - \hat{r}_{St})$ satisfies

$$E_t[\beta^{-1}\hat{\phi}_{t+1}(\hat{r}_{Lt+1} - \hat{r}_{St+1})] = 0 + O(\epsilon^3),$$

i.e. the realized excess return on the time-varying element of the portfolio is a zero-mean i.i.d. random variable up to a second-order of accuracy. As we did for the first-order accurate model solution, we therefore initially treat this term as an exogenous zero mean i.i.d. random variable ϵ_{0t}. We thus use the budget constraint with $\hat{\xi}_t$ in place of the excess return term and append to the other model equilibrium conditions the following equation:

$$E_t\hat{\xi}_{t+1} = \phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) + \phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2).$$

The second-order accurate solution of the model using the SGU algorithm is given by

$$\begin{aligned}
[x']^i &= [h_x]^i_a[x]^a + \frac{1}{2}[h_{xx}]^i_{ab}[x]^a[x]^b + \frac{1}{2}[h_{\sigma\sigma}]^i\sigma^2 + \sigma[\eta]^i_c[\epsilon']^c + O(\epsilon^3), \\
[y]^i &= [g_x]^i_j[x]^j + \frac{1}{2}[g_{xx}]^i_{jk}[x]^j[x]^k + \frac{1}{2}[g_{\sigma\sigma}]^i\sigma^2 + O(\epsilon^3).
\end{aligned}$$

Forwarding the solution for control variables one period ahead, substituting the solution for contemporaneous state variables and distinguishing terms of first- and second-order for the state vector leads to

$$[y']^i = [D_0]^i + [D_1]^i_a([x^f]^a + [x^s]^a) + [D_2]^i_c[\epsilon']^c + [D_3]^i_{ab}[x]^a[x]^b + [D_4]^i_{cd}[\epsilon']^c[\epsilon']^d + [D_5]^i_{ac}[x]^a[\epsilon']^c + O(\epsilon^3),$$

where the arrays $[D_0]$, $[D_1]$, $[D_2]$, $[D_3]$, $[D_4]$, $[D_5]$ are some functions of the arrays $[h_x]$, $[h_{xx}]$, $[h_{\sigma\sigma}]$, $[g_x]$, $[g_{xx}]$ and $[g_{\sigma\sigma}]$. We can thus write the second-order accurate conditional solution for the difference in marginal utility $(\sigma\hat{C}' - \sigma\hat{c}')$ and the excess return $(\hat{r}'_L - \hat{r}'_S)$ as

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [D_0]^1 + [D_1]^1_a([x^f]^a + [x^s]^a) + [D_2]^1_c[\epsilon']^c + [D_3]^1_{ab}[x^f]^a[x^f]^b \\
&\quad + [D_4]^1_{cd}[\epsilon']^c[\epsilon']^d + [D_5]^1_{ac}[x^f]^a[\epsilon']^c + O(\epsilon^3) \\
(\hat{r}'_L - \hat{r}'_S) &= [D_0]^2 + [D_1]^2_a([x^f]^a + [x^s]^a) + [D_2]^2_c[\epsilon']^c + [D_3]^2_{ab}[x^f]^a[x^f]^b \\
&\quad + [D_4]^2_{cd}[\epsilon']^c[\epsilon']^d + [D_5]^2_{ac}[x^f]^a[\epsilon']^c + O(\epsilon^3)
\end{aligned}$$

Now, we again split ϵ into ϵ_0 and $\tilde{\epsilon}$, and define $[D_2^0]^i \equiv [D_2]^i_1$, $[\tilde{D}_2]^i_c \equiv [D_2]^i_{c+1}$, $[\tilde{D}_4]^i_{cd} \equiv [D_4]^i_{c+1,d+1}$ and $[\tilde{D}_5]^i_{ac} \equiv [D_5]^i_{a,c+1}$. Note that second-order terms that contain ϵ'_0 disappear from the second-order accurate expressions (i.e. are part of the $O(\epsilon^3)$ term) since ϵ_0 is actually itself a second-order term. Hence, we can write the two expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ as

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [D_0]^1 + [D_1]^1_a([x^f]^a + [x^s]^a) + [D_2^0]^1\epsilon'_0 + [\tilde{D}_2]^1_c[\tilde{\epsilon}']^c + [D_3]^1_{ab}[x^f]^a[x^f]^b \\
&\quad + [\tilde{D}_4]^1_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + [\tilde{D}_5]^1_{ac}[x^f]^a[\tilde{\epsilon}']^c + O(\epsilon^3), \qquad \text{(C.17)} \\
(\hat{r}'_L - \hat{r}'_S) &= [D_0]^2 + [D_1]^2_a([x^f]^a + [x^s]^a) + [D_2^0]^2\epsilon'_0 + [\tilde{D}_2]^2_c[\tilde{\epsilon}']^c + [D_3]^2_{ab}[x^f]^a[x^f]^b \\
&\quad + [\tilde{D}_4]^2_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + [\tilde{D}_5]^2_{ac}[x^f]^a[\tilde{\epsilon}']^c + O(\epsilon^3) \qquad \text{(C.18)}
\end{aligned}$$

Since up to a first-order, the expected excess return is zero, we know that $[D_1]^2_a[x^f]^a = 0$. Further, since up to a second-order, the expected excess return is constant, we know that $[D_1]^2_a[x^s]^a$ and $[D_3]^2_{ab}[x^f]^a[x^f]^b$ are constants. Taking expectations on both sides of (C.18) implies that

$$[D_0]^2 = E[(\hat{r}'_L - \hat{r}'_S)] - [D_1]^2_a[x^s]^a - [D_3]^2_{ab}[x^f]^a[x^f]^b - [\tilde{D}_4]^2_{cd}[\Sigma]^{cd}$$

so that $(\hat{r}'_L - \hat{r}'_S)$ can be written as

$$(\hat{r}'_L - \hat{r}'_S) = E[(\hat{r}'_L - \hat{r}'_S)] - [\tilde{D}_4]^2_{cd}[\Sigma]^{cd} + [D_2^0]^2\epsilon'_0 + [\tilde{D}_2]^2_c[\tilde{\epsilon}']^c + [\tilde{D}_4]^2_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + [\tilde{D}_5]^2_{ac}[x^f]^a[\tilde{\epsilon}']^c + O(\epsilon^3)$$

We now recognize that ϵ'_0 is endogenous and given by

$$\epsilon'_0 = \beta^{-1}\hat{\phi}'(\hat{r}'_L - \hat{r}'_S) = \beta^{-1}[\psi]_k[\tilde{D}_2]^2_c[x^f]^k[\tilde{\epsilon}']^c$$

and use this to rewrite the second-order accurate expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ as

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [D_0]^1 + [D_1]^1_a([x^f]^a + [x^s]^a) + [\tilde{D}_2]^1_c[\tilde{\epsilon}']^c + [D_3]^1_{ab}[x^f]^a[x^f]^b \\
&\quad + [\tilde{D}_4]^1_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + \left([\tilde{D}_5]^1_{kc} + [D_2^0]^1\beta^{-1}[\tilde{D}_2]^2_c[\psi]_k\right)[x^f]^k[\tilde{\epsilon}']^c + O(\epsilon^3), \\
(\hat{r}'_L - \hat{r}'_S) &= E[(\hat{r}'_L - \hat{r}'_S)] - [\tilde{D}_4]^2_{cd}[\Sigma]^{cd} + [\tilde{D}_2]^2_c[\tilde{\epsilon}']^c \\
&\quad + [\tilde{D}_4]^2_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + \left([\tilde{D}_5]^2_{kc} + [D_2^0]^2\beta^{-1}[\tilde{D}_2]^2_c[\psi]_k\right)[x^f]^k[\tilde{\epsilon}']^c + O(\epsilon^3).
\end{aligned}$$

The first-order accurate solutions for $\hat{C}'$, $\hat{c}'$, $\hat{r}'_L$, $\hat{r}'_S$ and $\hat{q}_L$ are also useful:

$$\begin{aligned}
\hat{C}' &= [D_1]^3_a[x^f]^a + [\tilde{D}_2]^3_c[\tilde{\epsilon}']^c + O(\epsilon^2), \\
\hat{c}' &= [D_1]^4_a[x^f]^a + [\tilde{D}_2]^4_c[\tilde{\epsilon}']^c + O(\epsilon^2), \\
\hat{r}'_L &= [D_1]^5_a[x^f]^a + [\tilde{D}_2]^5_c[\tilde{\epsilon}']^c + O(\epsilon^2), \\
\hat{r}'_S &= [D_1]^6_a[x^f]^a + [\tilde{D}_2]^6_c[\tilde{\epsilon}']^c + O(\epsilon^2), \\
\hat{q}_L &= [h_x]^4_a[x^f]^a + O(\epsilon^2),
\end{aligned}$$

where we note that $[D_1]^5_a = [D_1]^6_a$.

Combining the third-order approximations to the household's and entrepreneur's portfolio equations yields:

$$\begin{aligned}
&E\left[\left(\sigma\hat{C}' - \sigma\hat{c}'\right)(\hat{r}'_L - \hat{r}'_S) - \frac{1}{2}\left(\sigma^2(\hat{C}')^2 - \sigma^2(\hat{c}')^2\right)(\hat{r}'_L - \hat{r}'_S) + \frac{1}{2}\left(\sigma\hat{C}' - \sigma\hat{c}'\right)\left((\hat{r}'_L)^2 - (\hat{r}'_S)^2\right)\right] \\
&= -(1-\beta)\sigma^2_\vartheta + \tau^S + (1-\beta)\sigma^2_\vartheta\hat{q}_L + (1-\beta)\sigma^2_\vartheta\sigma E\hat{C}' + \tau^S\hat{r}'_S - \tau^S E\hat{c}' + O(\epsilon^4)
\end{aligned}$$

We assume that all third moments of the vector of exogenous shocks $\tilde{\epsilon}'$ are zero. Substituting the expressions derived above for $(\sigma\hat{C}' - \sigma\hat{c}')$, $(\hat{r}_L - \hat{r}_S)$, $\hat{C}'$, $\hat{c}'$, $\hat{r}'_L$, $\hat{r}'_S$ and $\hat{q}_L$ into the terms of the third-order portfolio equation leads, after a number of manipulations and simplifications, to a zero-order accurate expression for the coefficients of the assumed linear decision rule for $\hat{\phi}_t$:

$$\psi_a = -\beta\frac{[\tilde{D}_5]^1_{ac}[\tilde{D}_2]^2_d[\Sigma]^{cd} + [\tilde{D}_5]^2_{ad}[\tilde{D}_2]^1_c[\Sigma]^{cd} - (1-\beta)\sigma^2_\vartheta\left([D_1]^5_a + [h_x]^4_a\right)}{[D_2^0]^1[\tilde{D}_2]^2_c[\tilde{D}_2]^2_d[\Sigma]^{cd} - (1-\beta)\sigma^2_\vartheta[D_2^0]^2 + \tau^S[D_2^0]^2} + O(\epsilon)$$

For $\sigma_\vartheta = \tau^S = 0$, this expressions collapses to the one in Devereux and Sutherland (2009).

Second-order solution for non-portfolio variables, once first-order ϕ_t is known

Once the ψ_a's have been computed, we can go back to the original model and replace ϕ_t by

$$\phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) + \frac{1}{2}\phi\beta^{-1}(\hat{r}^2_{Lt} - \hat{r}^2_{St}) + \beta^{-1}\left([\psi]_k[x_{t-1}]^k\right)(\hat{r}_{Lt} - \hat{r}_{St})$$

in the entrepreneur's budget constraint, and replace ϕ_t by ϕ in the economy's resource constraint. Note that again, these are not a correct equilibrium condition, but just an artifice to solve the model using the SGU algorithm. Since taking a second-order approximation of the modified entrepreneur's budget constraint gives the same expression as a second-order approximation of the true budget constraint and taking a second-order approximation of the modified resource constraint gives the same expression as the second-order approximation of the true resource constraint, as far as second-order accuracy is concerned, we can solve the model using these modified constraints.