Stagflation and Topsy-Turvy Capital Flows[*]

Julien Bengui
Bank of Canada
and CEPR

Louphou Coulibaly
University of Wisconsin-Madison
and NBER

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Abstract

Should we expect financial openness to facilitate macroeconomic adjustment in a context of heightened stagflation concerns? Using an open-economy model with nominal rigidities, we argue that, quite to the contrary, free capital mobility undermines stabilization policy when the latter faces an output-inflation trade-off. Capital inflows cause unwelcome upward pressure on domestic marginal costs in high-inflation countries, thus deteriorating policy trade-offs. Yet, market forces are likely to generate inflows into these countries. A constrained efficient regime features flows in the opposite direction, suggesting *topsy-turvy* capital flows following supply shocks.

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Bengui: jbengui@bankofcanada.ca and Coulibaly: c.louphou@gmail.com

1 Introduction

One of the most striking and unexpected macroeconomic development of the ongoing recovery is the recent pick up in inflation. To prevent inflation from settling at high levels, major central banks are expected to tighten their monetary policy stance to varying degrees over the coming quarters. Cross-country heterogeneity in the pace of tightening is likely to lead to asynchronous economic slowdowns, which, in turn, can be expected to trigger adjustments of international relative prices and capital flows. A key question for international macroeconomics in this context is whether the free capital mobility system that most of the world economy has strove for over the last few decades can be expected to facilitate macroeconomic adjustments or hamper it.

The idea that financial openness facilitates an economy's short-term adjustment to macroeconomic shocks has a long history dating back to classical economics thinking and has, over time, permeated key global policy institutions such as the International Monetary Fund (IMF) and the World Bank (see, e.g., Williamson 1990). Over the last two decades, however, both the economics literature and policy views have evolved into a more nuanced stance. On the one hand, the main institutional proponents of the Washington consensus have come to acknowledge that measures that limit short-term capital flows can be useful in some circumstances (IMF 2012). On the other hand, advances in the macroeconomic theory literature have increasingly pointed to imperfections in financial, goods and labor markets as possible causes of excessive capital flows (e.g., Bianchi 2011 and Schmitt-Grohe and Uribe 2016). Yet, perhaps because adverse supply shocks have been off policymakers' radar for a while, no study has explicitly considered the role of output-inflation trade-offs to assess the desirability of financial openness and cross-country capital mobility. Our goal in this paper is to focus on this issue.

We point to a novel pecuniary externality associated with external borrowing and operating through the economy's supply side: External borrowing shifts up labor supply and, under fairly mild conditions, raises domestic firms' marginal costs. When the economy operates at potential and inflation is perfectly stabilized, this externality does not generate first-order welfare effects and, hence, does not cause inefficiencies. However, when the economy operates below potential as a result of the central bank's attempt to fight off a cost-push shock (i.e., in a stagflation scenario), the rise in marginal costs worsens the policy trade-off: To stabilize inflation at a given level, the central bank needs to engineer a more severe recession. In this case, the externality does generate first-order welfare effects and creates a wedge between the privately and socially optimal levels of external borrowing.

We formalize this insight in a simple open-economy general equilibrium model with nominal rigidities, whose ingredients form the backbone of more elaborate dynamic stochastic general equilibrium (DSGE) models used by most central banks for policy analysis. We question the constrained efficiency of external borrowing decisions in that context. That is, assuming that labor supply, expenditure allocation and price setting decisions are made by individual households and firms, we ask whether a planner would choose the same level of external saving or borrowing as private households.

We find that a regime of free capital mobility is constrained inefficient in the presence of an output-inflation trade-off due to the externality described above. Firms' marginal costs can be decomposed into a pure labor cost component, depending on the real wage measured in terms of the economy's consumption basket, and an adjustment term accounting for the economy's relative purchasing power. For a given output gap, a marginal increase in external borrowing raises domestic spending, shifting up labor supply and causing a rise in the equilibrium real wage. When the model features home bias in consumption, the rise in domestic spending also appreciates the terms of trade, which raises the purchasing power of domestic firms and attenuates the rise in their marginal costs. When the trade elasticity is very low, this latter effect on purchasing power overwhelms the effect on labor costs and leads to an overall drop in firms' marginal costs. But for plausible calibrations of elasticities, the labor cost channel dominates and more external borrowing raises marginal costs.

If the Home and Foreign economies face output-inflation trade-offs of identical stringency, i.e., if an optimal policymaker's multipliers on their Phillips curves are equal, this pecuniary externality does not induce inefficient borrowing. But if the output-inflation trade-off is more stringent in one of the two countries, then households in the country where inflation is the highest overborrow, and an optimal capital flow management policy would call for redirecting spending away from the country with the most negative output gap.[1] Intuitively, when monetary policy already faces an unfavorable trade-off and adopts a particularly tight stance to limit domestic inflation at the expense of undesirably low economic activity, additional capital inflows make monetary policy's job even harder. Either the central bank lets the rise in marginal costs translate into higher domestic inflation, or it is forced to depress economic activity further to achieve a given stabilization of inflation. Either way, the economy is worse off, and this adverse side effect of external borrowing is not adequately signaled by its price in an unregulated market.

[1] In our model, such cross-country differences in the stringency of output-inflation trade-offs are the result of asymmetric cost-push shocks, but more generally, they could arise from any structural asymmetry across countries.

The externality we point to does not simply lead to inefficiencies at the margin. Indeed, it is powerful enough to reverses the direction of capital flows in response to cost-push shocks. While a free capital mobility regime is likely to feature capital inflows into the region with the deepest recession, a constrained efficient regime always prescribes outflows from that region. Our analysis hence suggests that ostensibly wrong price signals in international financial markets can lead to *tospy-turvy* capital flows during a stagflation episode.

Our externality resembles those stressed by two branches of the recent literature in monetary and international macroeconomics. In the first one, elegantly exposed in general terms by Farhi and Werning (2016), privately optimal financial choices differ from socially optimal ones due to aggregate demand (AD) externalities in economies with nominal rigidities.[2] In the second one, following Caballero and Krishnamurthy (2001), pecuniary externalities generate inefficiencies in incomplete markets environments.[3] The frictions driving inefficiencies in our model – monopolistic competition and nominal rigidities in goods market – correspond more closely to those emphasized by the first branch. At the same time, the type of externality we document works more similarly to the one studied in the second branch. Indeed, rather than affecting other agents through aggregate demand directly, in our model households impose externalities on others through two key market prices, i.e., the real wage and the terms of trade. This is similar to the pecuniary externality literature. But in our work, the pecuniary externality has welfare ramifications because the economy finds itself away from the first-best allocation due to monetary policy's inability to simultaneously stabilize inflation and economic activity, rather than due a failure to share risk or trade inter-temporally as in the existing pecuniary externality literature (e.g., Davila and Korinek 2017).

The contrast between the pecuniary externality we focus on and the AD externalities studied by Farhi and Werning (2016) and others goes beyond simple semantics. When AD externalities lead to inefficient financial decisions in contexts where constraints on price adjustment and monetary policy prevent goods-specific labor wages to be closed, the general prescription for taxes on financial transactions is to incentivize agents to shift

[2]See also Farhi and Werning (2012, 2014, 2017), Korinek and Simsek (2016), Schmitt-Grohe and Uribe (2016), Acharya and Bengui (2018), Fornaro and Romei (2019) and Bianchi and Coulibaly (2021).

[3]For earlier articulations of these ideas in the information economics and general equilibrium literatures, see, e.g., Stiglitz (1982), Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986). In financial economics, see, e.g., Gromb and Vayanos (2002) and Lorenzoni (2008). In international macroeconomics, see Korinek (2007, 2018), Bianchi (2011), Jeanne and Korinek (2010, 2019, 2020), Benigno, Chen, Otrok, Rebucci and Young (2013, 2016), Bengui (2014) and Bianchi and Mendoza (2018). Also, see Coulibaly (2020) and Ottonello (2021) for examples of studies combining pecuniary externalities mattering due to financial frictions with AD externalities arising from nominal rigidities.

wealth toward states of nature where their spending is relatively high on goods whose provision is most depressed. Intuitively, boosting spending on these goods is something monetary policy would like to do, but is unable to, due to constraints such as a fixed exchange rate (Farhi and Werning 2012, 2017, Schmitt-Grohe and Uribe 2016) or a zero lower bound (Farhi and Werning 2016, Korinek and Simsek 2016). This general principle does not apply in our context, where, for plausible calibration of elasticities, it is optimal to tilt spending *away* from the country whose output gap is the most negative. In our model, in response to an inflationary cost-push shock, output is depressed not because monetary policy lacks the mean to prop it up, but rather because monetary policy chose to engineer a recession in order to control inflation. As a result, financial market interventions should incentivize agents to shift wealth away from states of nature where their spending worsens the least favorable output-inflation trade-off. Our paper hence complements the AD externality literature by providing an insight specific to circumstances where, as at the current juncture, economic slowdowns may be triggered by central banks' desire to limit inflation.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 characterizes optimal monetary and capital flow management policy, establishing the constrained inefficiency of the free capital mobility regime. Section 4 makes our results more tangible by studying the world economy's adjustment to a shock generating an output-inflation trade-off and Section 5 concludes.

2 Model

The world is composed of two country of equal size, Home and Foreign. In each country, households consume goods and supply labor, while firms hire labor to produce output. Variables pertaining to Foreign are denoted with asterisks.

2.1 Households

In each country, there is a representative household. In the home country, the preferences of the representative household are represented by the utility functional:[4]

$$\int_0^\infty e^{-\rho t}\left[\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\phi}}{1+\phi}\right]dt,$$

[4]Our model exposition focuses on households in Home, but households in Foreign are symmetric.

where C_t is consumption, N_t is labor supply, ϕ is the inverse Frisch elasticity of labor supply, σ is the inverse of the inter-temporal elasticity of substitution, and ρ is the discount rate. The consumption index C_t is defined as

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

In turn, $C_{H,t}$ and $C_{F,t}$ are CES aggregates over a continuum of goods produced respectively in Home and Foreign, with elasticity of substitution between varieties produced within a region equal to $\varepsilon > 1$. The elasticity of substitution between domestic and foreign goods is $\eta > 0$ and $\alpha \in (0, 1/2]$ is a home bias parameter capturing the degree of openness. When $\alpha = 1/2$, there is no home bias as households in the home country and the foreign country consume the same basket of goods. In contrast, when $\alpha < 1/2$, there is home bias in consumption as households value more highly domestic goods.

In each country, an household can trade two types of bonds in credit markets: an international nominal bond B_t and a domestic nominal bond denoted D_t in Home and D_t^* in Foreign that can be traded only among domestic households. The international bond is (arbitrarily) denominated in the home currency, without loss of generality.

The household's budget constraint in the home country is given by:

$$\dot{D}_t + \dot{B}_t = i_t D_t + i_{B,t} B_t + W_t N_t + \Pi_t - \int_0^1 P_{H,t}\left(l\right) C_{H,t}(l) dl - \int_0^1 P_{F,t}(l) C_{F,t}(l) dl$$

where W_t is the nominal wage, Π_t is the payout of domestic firms, i_t denotes the return on Home bonds and $i_{B,t}$ denotes the return on the international claims held by Home households.

Foreign households are symmetric. Foreign households and Home households are similar as far as preferences toward consumption and leisure where as noted above Foreign variables are indexed by asterisks. We assume that the return on international claims held by home households and foreign households has two components: a component that is common across countries $\underline{i}_t$ and a country-specific component (τ_t for Home and τ_t^* for Foreign) that captures financial regulations imposed by a global financial regulator on international borrowing. We denote by ξ_t the wedge between the return on international bond faced by households in the two countries

$$\xi_t \equiv i_{B,t} - i_{B,t}^* = \tau_t - \tau_t^*. \tag{1}$$

With frictionless international asset market $\xi_t = 0$ for all $t \geq 0$. We assume that countries have symmetric net foreign asset positions (i.e., equal to 0) at time 0.

Standard expenditure minimization leads to consumer price indices (CPI) in Home and Foreign given by

$$P_t \equiv \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{1/(1-\eta)},$$
$$P_t^* \equiv \left[(1-\alpha)(P_{F,t}^*)^{1-\eta} + \alpha(P_{H,t}^*)^{1-\eta}\right]^{1/(1-\eta)},$$

$P_{H,t}$ (and $P_{F,t}^*$) being the Home's (and Foreign's) PPI and $P_{F,t}$ (and $P_{H,t}^*$) being Home's (and Foreign's) price index of imported goods. The terms of trade between the Home and Foreign are defined as the ratio of PPIs, $S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \frac{P_{F,t}^*}{P_{H,t}^*}$, while the real exchange rate is defined as the ratio of CPIs expressed in a common currency, $Q_t \equiv \frac{E_t P_t^*}{P_t}$, where E_t is the nominal exchange rate.

Households in each country choose consumption, labor supply and bond holdings to maximize utility. Their optimality conditions for labor supply and domestic bond holdings in log-linearized form are given by

$$w_t - p_t = \phi n_t + \sigma c_t, \tag{2a}$$
$$w_t^* - p_t^* = \phi n_t^* + \sigma c_t^*, \tag{2b}$$
$$\sigma \dot{c}_t = i_t - \pi_t - \rho, \tag{3a}$$
$$\sigma \dot{c}_t^* = i_t^* - \pi_t^* - \rho, \tag{3b}$$

where lower case letters denote logarithms of the respective capital letter variables, $\pi_t \equiv \dot{P}_t / P_t$ is the Home CPI inflation and $\pi_t^* \equiv \dot{P}_t^* / P_t^*$ is the Foreign CPI inflation. (2a) and (2b) are the optimality conditions for households choice of labor supply which equate the marginal disutility of work to the real wage. (3a) and (3b) are the Euler equation for domestic bonds. The remaining conditions are no arbitrage conditions between domestic bonds and international bonds, $i_t = i_{B,t}$ and $i_t^* = i_{B,t}^* - \dot{e}_t$ which combined leads to the following distorted interest parity condition,

$$i_t = i_t^* + \dot{e}_t + \xi_t.$$

2.2 Firms

Technology. Firms in Home and in Foreign produce differentiated goods $l \in [0,1]$ with a linear technology: $Y_t(l) = AN_t(l)$, resp. $Y_t^*(l) = A^* N_t^*(l)$, where A and A^* denote productivity parameters normalized to one for convenience. As in Engel (2011), we let $N_t(l)$ (resp. $N_t^*(l)$) be a composite of individual household labor in Home (resp. Foreign) using a constant elasticity of substitution aggregator, where the elasticity of substitution among varieties of domestic labor in each country, ε_t^w (resp. ε_t^{w*}), is stochastic and common to all firms within the country. The variation in wage markups, $\mu_t^w \equiv \frac{\varepsilon_t^w}{\varepsilon_t^w - 1}$ and $\mu_t^{w*} \equiv \frac{\varepsilon_t^{w*}}{\varepsilon_t^{w*}-1}$, are the sources of cost-push shocks that give rise to well-known trade-offs between achieving a zero output gap and stabilizing inflation (see e.g., Clarida, Gali and Gertler, 2002).

Price setting. Firms, which operate under monopolistic competition, engage in infrequent price setting à la Calvo (1983). Each firm has an opportunity to reset its prices when it receives a price-change signal, which itself follows a Poisson process with intensity $\rho_\delta \geq 0$. As a result, a fraction δ of firms receives a price-change signal per unit of time. These firms reset their price, $P_{H,t}^r(j)$, to maximize the expected discounted profits

$$\int_t^\infty \rho_\delta e^{-\rho_\delta (k-t)} \frac{\lambda_k}{\lambda_t} \left[P_{H,t}^r(j) - P_{H,k} MC_k \right] Y_{k|t} dk,$$

subject to the demand for their own good, $Y_{k|t} = \left(P_{H,t}^r / P_{H,k} \right)^{-\varepsilon} Y_k$, taking as given the paths of output in the home country Y, the Home PPI P_H, and the real marginal cost MC. The real marginal cost is defined as $MC_k \equiv (1-\tau^N) W_k / (A_k P_{H,k})$, where τ^N is a time-invariant labor subsidy.[5] λ_k denotes Home household's time k marginal utility of consumption, so that the ratio λ_k / λ_t is the firm's relevant discount factor between time t and time k. The pricing environment is symmetric in the foreign country. In the limiting case of flexible prices (i.e. $\rho_\delta \to \infty$), firms are able to reset their prices continuously and optimal pricing setting reduces to $P_{H,t} = (1-\tau^N) \frac{\varepsilon}{\varepsilon - 1} W_t$.

2.3 Policy

The global planner sets the cooperative monetary policy by choosing the nominal interest rates i_t and i_t^* on domestic bonds in both countries. She also controls the relative wealth

[5] As is standard in the New Keynesian literature, we assume that this subsidy is set at the level that would be optimal in a steady state with flexible prices. This subsidy can thus be thought of as offsetting long-run distortions stemming from monopolistic competition.

$\{\Theta_t\}_{t\geq 0}$ by introducing distortionnary financial regulations in the international asset market, i.e. setting the path for $\{\xi_t\}_{t\geq 0}$, and determines the date 0 transfer $\mathcal{T}_0$ from Foreign to Home consistent with the chosen path for $\{\Theta_t\}_{t\geq 0}$,

$$\mathcal{T}_0 = \int_0^\infty e^{-\rho t} \left(C_t^*\right)^{1-\sigma} Q_t^{\frac{1}{\sigma}-1} \left[\Theta_t - \left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} \left((1-\alpha)\Theta_t + \alpha Q_t^{\eta-1}\right)\right] dt,$$

to maximize global welfare. The global planner sets of policy instruments is $\{i_t, i_t^*, \Theta_t, \mathcal{T}_0\}$.

2.4 Equilibrium Dynamics

Given a specification of monetary and capital flow management policy, an equilibrium is a constellation where all households and firms optimize while markets clear.

International consumption smoothing. Combining the Home household's Euler equation with its Foreign household's counterpart for the international bonds gives an international consumption smoothing condition relating the ratio of marginal utility in both countries to the real exchange rate[6]

$$C_t = \Theta_t Q_t^{\frac{1}{\sigma}} C_t^*, \tag{4}$$

where $\Theta_t \equiv \Theta_0 \exp\left[\frac{1}{\sigma}\int_0^t \xi_s ds\right]$, with Θ_0 being a constant related to initial relative wealth positions. Given our assumption of symmetric initial positions, condition (4) indicates that the marginal utility of nominal wealth for Home and Foreign households are equalized when international bond markets are frictionless. By controlling for Θ_t, the global regulator indirectly controls for international demand imbalances and thus capital flows across countries. The global planner's policy instruments and objective are described in section 2.3. Taking logs on both sides of (4), and taking into account the (first-order accurate) relationship between the real exchange rate and the terms of trade, $q_t = (1-2\alpha)s_t$, we obtain the log-linearlized international consumption smoothing condition

$$\sigma(c_t - c_t^*) = \theta_t + (1-2\alpha)s_t. \tag{5}$$

[6] In models featuring uncertainty and complete markets, this condition is often labeled as an international risk sharing condition. Notice that (4) bears similarity to what is commonly referred to as the Backus-Smith condition (see Kollmann 1991 and Backus and Smith 1993) in which Θ_t would represent a Pareto weight in a planning problem.

Output determination. Market clearing for a good l produced in Home requires that the supply of the good equals the sum of the demand emanating from Home and Foreign:

$$Y_t(l) = \underbrace{(1-\alpha)\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t}_{C_{H,t}(l)\text{: Home demand for Home variety } l} + \underbrace{\alpha\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*}_{C_{H,t}^*(l)\text{: Foreign demand for Home variety } l} . \tag{6}$$

At the level of Home's aggregate output, market clearing requires

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)\,C_t + \alpha Q_t^{\eta} C_t^*\right].$$

A first order approximation of this condition around the symmetric steady state yields the following log-linear expression:

$$y_t = (1-\alpha)\left[c_t + \alpha\eta s_t\right] + \alpha\left[c_t^* + (1-\alpha)\eta s_t\right]. \tag{7a}$$

Similarly, the log-linearized Foreign goods market clearing condition is given by

$$y_t^* = (1-\alpha)\left[c_t^* - \alpha\eta s_t\right] + \alpha\left[c_t - (1-\alpha)\eta s_t\right]. \tag{7b}$$

These expressions indicate that output in each country depends on consumption in Home and Foreign, as well as on the terms of trade: a terms of trade improvement for Home (i.e., a decrease in s_t) raises output in Foreign at the expense of output in the core via the expenditure switching channel.

Combining the consumption smoothing relation (5) with the market clearing conditions (7a) and (7b) yields an expression for the equilibrium terms of trade:

$$\sigma(y_t - y_t^*) = \omega s_t + (1-2\alpha)\sigma\theta_t, \tag{8}$$

for $\omega \equiv \sigma\eta - (\sigma\eta - 1)(1-2\alpha)^2 > 0$. The expression indicates that output is relatively higher in the country which has less favorable terms of trade or, in the presence of home bias in consumption, in the country benefiting from a demand imbalance. In the absence of home bias (i.e., when $\alpha = 1/2$), since the composition of consumption is identical across the two countries, (consumption) demand imbalances do not translate into output differences. Combining the budget constraints of households, firms, as well as the condition relating the equilibrium terms of trade and the relative output (8), we arrive at the trade balance condition:

$$nx_t = \frac{\omega - 1}{2\sigma} s_t - \alpha\theta_t. \tag{9}$$

which says that the effects of an appreciated terms of trade on the trade balance depends on the relative importance of the elasticities of substitution across goods (η) and across time ($1/\sigma$). Furthermore, a positive demand imbalance ($\theta_t > 0$) is associated with capital inflows (i.e., negative net exports).

Denoting aggregate output in the home country as $Y_t \equiv \left[\int_0^1 Y_t(l)^{(\varepsilon-1)/\varepsilon} dl\right]^{\varepsilon/(\varepsilon-1)}$, aggregate employment relates to aggregate output according to $N_t \equiv \int_0^1 N_t(l)dl = Y_t Z_t$, where $Z_t \equiv \int_0^1 (P_t(l)/P_t)^{-\varepsilon} dl$. Since equilibrium variations in $z_t \equiv \ln Z_t$ around the steady state are of second order, up to a first order approximation, the relationships between aggregate employment and output is given by:

$$n_t = y_t, \quad \text{and} \quad n_t^* = y_t^*. \tag{10}$$

Inflation and marginal costs. Under our Calvo price setting assumption, up to a first-order approximation, the dynamics of PPI inflation in terms of the real marginal cost in each region are described by

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \kappa\widehat{mc}_t, \tag{11a}$$

$$\dot{\pi}^*_{F,t} = \rho\pi^*_{F,t} - \kappa\widehat{mc}^*_t. \tag{11b}$$

where $\kappa \equiv \rho_\delta(\rho + \rho_\delta)$, and $\widehat{mc}_t$ (resp. $\widehat{mc}^*_t$) denotes the log deviation of the real marginal cost from its steady state value. Using the aggregate production functions (10) and the labor supply equations (2a), these are given by

$$\widehat{mc}_t = (\sigma + \phi)y_t - \frac{\omega - 1}{2} s_t + \alpha\sigma\theta_t + u_t, \tag{12a}$$

$$\widehat{mc}^*_t = (\sigma + \phi)y^*_t + \frac{\omega - 1}{2} s_t - \alpha\sigma\theta_t + u^*_t. \tag{12b}$$

Intuitively, the real marginal cost (measured in units of the domestic good) depends negatively on productivity, positively on the marginal rate of substitution between consumption and leisure and negatively on the terms of trade.[7] However, since the equilibrium marginal

[7]That is to say, an improvement in a country's terms of trade lowers its producers' marginal cost. A terms of trade improvement raises the price of the domestic good relative to that of the consumption basket. Noting that $p_t = p_{H,t} + \alpha s_t$, the labor supply equation (2a) implies that the real wage expressed in terms of the domestic good must be equal to $w_t - p_{H,t} = \phi n_t + \sigma c_t + \alpha s_t$, so that the real marginal cost is given by $\widehat{mc}_t = \phi n_t + \sigma c_t - a_t + \alpha s_t$.

rate of substitution itself depends ambiguously on the terms of trade (controlling for output and relative consumption), the relationship between the terms of trade and the marginal cost is a priori ambiguous. Finally, controlling for output and the terms of trade, higher relative consumption in a country raises its residents' marginal rate of substitution and thus increases the marginal cost.[8] The cost-push shocks, $u_t \equiv \mu_t^w - \mu^w$ and $u_t^* \equiv \mu_t^{w*} - \mu^w$, are deviations of wage markups from their steady state value.

2.5 World and Difference formulation

Before studying the optimal policy response to asymmetric cost-push shocks, it is convenient to rewrite the dynamics of output and inflation in both regions in "world" and "difference" format. We define the world output gap and the cross-country output gap differential as $y_t^W = (y_t + y_t^*)/2$ and $y_t^D = (y_t - y_t^*)/2$. Similarly, we define the world PPI inflation and cross-country PPI inflation differential as $\pi_t^W = (\pi_{H,t} + \pi_{F,t}^*)/2$ and $\pi_t^D = (\pi_{H,t} - \pi_{F,t}^*)/2$. Combining PPI inflation dynamics (11a)-(11b) in gaps yields both the world New-Keynesian Phillips curve and the New-Keynesian Phillips curve (NKPC) in difference

$$\dot{\pi}_t^W = \rho\pi_t^W - \kappa(\sigma + \phi)y_t^W - \kappa u_t^W, \tag{13}$$

$$\dot{\pi}_t^D = \rho\pi_t^D - \kappa\left[(\sigma + \phi)y_t^D - \frac{\omega - 1}{2}s_t + \alpha\sigma\theta_t\right] - \kappa u_t^D. \tag{14}$$

We also note that the equilibrium terms of trade expression (8) can be written as

$$2y_t^D = \frac{\omega}{\sigma}s_t + (1 - 2\alpha)\theta_t. \tag{15}$$

This relationship reveals that a more positive output gap in Home than in Foreign can arise for two reasons. On the one hand, an appreciated terms of trade $s_t < 0$ shifts demand away from Home goods towards Foreign goods, leading to a negative output gap differential. On the other hand, to the extent that there is some home bias in preferences ($\alpha < 1/2$), a positive demand gap raises demand more of the Home good than for the foreign good.

[8] Note that for Home, improved terms of trade correspond to a lower s_t while a higher relative consumption corresponds to a higher θ_t. In contrast, for Foreign, improved terms of trade correspond to a higher s_t while a higher relative consumption corresponds to a lower θ_t.

3 Characterization of optimal policy

We start by characterizing optimal monetary and capital flow management policy in the model just presented. We show that when monetary policy faces an output-inflation trade-off, a free capital mobility regime is generically constrained inefficient due to two distinct pecuniary demand externalities operating via firms' marginal costs. For plausible calibrations of the model, the region experiencing higher inflation overborrows and capital flows into rather than out of the region with the deepest recession.

3.1 Welfare-based loss function

To capture the various trade-offs to be resolved by optimal monetary and capital flow management policies, we use a standard welfare-based loss function. To obtain this loss function, we take a second-order approximation of a symmetrically weighted average of households' utilities in Home and Foreign (see Appendix A).[9] The instantaneous loss function is given by

$$L_t = \left[(\sigma+\phi)(y_t^W)^2 + \frac{\varepsilon}{\kappa}(\pi_t^W)^2\right] + \left[(\sigma+\phi)(y_t^D)^2 + \frac{\varepsilon}{\kappa}(\pi_t^D)^2\right] \\ + \alpha(1-\alpha)(1-\sigma\eta)\eta(s_t)^2 + \sigma\alpha(1-\alpha)\left[\theta_t - (\sigma\eta-1)(1-2\alpha)\sigma^{-1}s_t\right]^2 \quad (16)$$

where the output gap and inflation are again expressed in "world" and "difference" forms. The first two terms in (16) featuring squared output gaps and inflation reflect sticky price distortions familiar from the closed economy literature. The third and fourth terms, reflecting distortions specific to the open economy context, capture welfare losses stemming from an inefficient cross-country distribution of consumption potentially caused by two factors: the demand imbalance θ_t and the terms of trade gap s_t.[10]

Normative research in new open-economy macroeconomics (e.g., Benigno 2009) has not traditionally treated the demand imbalance term θ_t as a policy variable – e.g., instead setting it to zero under complete markets or determined by shocks and other macroeconomic variables under incomplete markets – and studied how monetary policy should be set optimally. Our approach, in contrast, is to ask whether actively managing the demand imbalance θ_t may be desirable in a context where it could otherwise be left at zero.

[9] Given equal country sizes, our adoption of equal welfare weight can be interpreted as an implicit assumption of perfect insurance with respect to the risk of the cost-push shock scenario described above.

[10] This later factor, however, disappears in a widely studied special case featuring unit elasticities (Cole and Obstfeld 1991), and more generally whenever the intra-temporal and inter-temporal elasticities are equal ($\eta = 1/\sigma$).

3.2 Optimal monetary policy

The optimal monetary policy problem consists in choosing a path for the welfare relevant output gaps y_t^W, y_t^D, inflation π_t^W, π_t^D, and terms of trade s_t, to minimize the present value of the loss (16), subject to the NKPCs (13), (14), and the equilibrium terms of trade expression (15).[11] We have the following characterization.

Proposition 1 (Optimal monetary policy)**.** *Optimal monetary policy is characterized by the following targeting rules:*

$$\dot{y}_t^W + \varepsilon \pi_t^W = 0 \tag{17}$$

$$\dot{y}_t^D + \varepsilon \pi_t^D = 0 \tag{18}$$

Proof. See Appendix B.1. □

This description of optimal cooperative monetary policy is analogous to that commonly encountered for complete markets open-economy models with producer currency pricing (PCP) in the literature. (17) and (18) indicate that, both in "world" and "difference" terms, optimal policy strikes a balance between losses from inflation and losses from deviations of output from its efficient level. The two targeting rules can be combined to deliver targeting rules for each country that only depend on the domestic output gap and inflation, i.e., $\dot{y}_t + \varepsilon \pi_{H,t} = 0$ and $\dot{y}_t^* + \varepsilon \pi_{F,t}^* = 0$, a feature referred to as *inward looking* monetary policy in the NOEM literature. It is worth stressing that this characterization does not rely on any particular assumption regarding the path of θ_t (other than it being exogenous, or chosen by policy). In particular, it holds under free capital mobility (i.e., $\theta_t = 0 \; \forall t$), as well as under an optimally managed capital account regime to be derived below.

The targeting rules (17) and (18) lead us to one observation, summarized in the corollary below, which helps us narrow down the role played by capital flows in response to shocks.

Corollary 1 (Irrelevance of capital flow regime for world variables)**.** *The paths of world output gap y_t^W and world inflation π_t^W are independent of the capital flow regime (i.e., of the path of θ_t).*

Proof. See Appendix B.2 □

[11] Implicitly, in line with the literature, we assume that the policymaker has access to a date 0 transfer so the optimal policy problem reflects efficiency rather than a mix of efficiency and redistributive considerations. For a formal statement of the optimal monetary policy problem, see Appendix B.1.

This observation follows directly from combining the "world" NKPC (13) with the "world" monetary policy targeting rule (17) and means that the capital flow regime only matters for the determination of cross-country "difference" variables and the terms of trade.[12] Therefore, both from a positive and normative standpoint, an analysis of the role played by capital flows in the adjustment to shocks can legitimately center on the dynamics of cross-country difference variables y_t^D, π_t^D and external variables s_t and θ_t.

Remark (Inward vs. outward looking monetary policy)**.** *When the path of the relative demand imbalance θ_t deviates from zero, asset markets are no longer complete and the inward lookingness of monetary policy in* (17)-(18) *contrasts with the* outward looking *rules derived in studies assuming other forms of market incompleteness, such as financial autarky or a single bond in environments featuring uncertainty (e.g., Corsetti, Dedola and Leduc 2010, 2018).*[13] *In these studies, the relative demand gap is an endogenous variable whose fluctuations depend on the interaction of shocks and other variables influenced by monetary policy (such as the cross-country difference in the output gap). As a result, monetary policy can manage distortions caused by market incompleteness and generally chooses to do so, resulting in outward looking rules. In our case, in contrast, the relative demand gap is either exogenous or directly controlled by policy, so there is no scope for monetary policy to manage market incompleteness distortions, hence the inward looking rules.*

3.3 Optimal capital flow management

To question the constrained efficiency of the free capital mobility regime, we make the demand imbalance θ_t a choice variable of the optimizing policy maker and ask under which circumstances θ_t is set to a value different from zero. The optimal policy problem now consists in choosing a path for the welfare relevant output gaps y_t^W, y_t^D, inflation π_t^W, π_t^D, terms of trade s_t *and* demand imbalance θ_t to minimize the present discounted value of the loss (16), subject to the NKPCs (13), (14) and the equilibrium terms of trade relation (15).[14]

In addition to the targeting rules associated with monetary policy, (17) and (18), optimal policy now also pertains to an additional capital flow management margin.

Proposition 2 (Optimal capital flow management)**.** *The optimal capital flow regime is charac-*

[12] See Groll and Monacelli (2020) for a similar result regarding the irrelevance of the exchange rate regime.

[13] The literature refers to outward looking monetary policy when targeting rules in open-economy models also feature external variables, such as international relative prices or the relative demand gap.

[14] See Appendix B.1 for a formal statement of the problem.

terized by the targeting rule

$$\theta_t = \frac{\sigma\chi - (1-2\alpha)}{\sigma\chi}\, 2y_t^D. \tag{19}$$

where $\chi \equiv 2(1-\alpha)\eta$ *denotes the trade elasticity.*[15]

Proof. See Appendix B.3 □

To the extent that shocks generating an output-inflation trade-off generally result in a non-zero cross-country difference in output gaps according to (14)-(15)-(18), the relative demand imbalance is only set to zero if $\sigma\chi = (1-2\alpha)$, or if the trade elasticity normalized by the intertemporal elasticity of substitution (IES) $1/\sigma$ is equal to the degree of home bias $1-2\alpha$. Away from this knife-edge case, the free capital mobility regime is constrained inefficient. To relate this inefficiency to the widely discussed notion of over-borrowing, it is convenient to consider a decentralization of the constrained efficient capital account regime via taxes on capital flows.

Decentralization with taxes on capital flows. Denoting by τ_t the tax on Home households' borrowing on the international market and by τ_t^* the tax on Foreign households' borrowing on that same market, the tax differential $\tau_t^D \equiv (\tau_t - \tau_t^*)/2$ can be derived from the consumption risk-sharing condition (4) as $\tau_t^D = \sigma\dot{\theta}_t/2$. The relationship between the optimal tax differential and the inflation differential can thus be obtained from combining the targeting rules (18)-(19) as

$$\tau_t^D = [\sigma\chi - (1-2\alpha)]\, \pi_t^D, \tag{20}$$

We therefore have the following characterization the free capital mobility regime.

Corollary 2 (Over- vs under-borrowing)**.** *The high-inflation country over-borrows when* $\sigma\chi > (1-2\alpha)$*, and under-borrows when* $\sigma\chi < (1-2\alpha)$*. The free capital mobility regime is only constrained efficient in non-generic cases where* $\sigma\chi = (1-2\alpha)$*.*

[15]The trade elasticity χ is defined as the sum of the absolute values of the price elasticity of imports and the price elasticity of exports, holding aggregate consumption constant. Formally,

$$\chi \equiv \left.\frac{-\partial \log C_{F,t}}{\partial \log P_{F,t}/P_{H,t}}\right|_{C_t} + \left.\frac{-\partial \log C^*_{H,t}}{\partial P^*_{H,t}/P^*_{F,t}}\right|_{C^*_t} = 2(1-\alpha)\eta.$$

Several insights emerge from Proposition 2 and Corollary 2. First, the free capital mobility regime is generically constrained inefficient when monetary policy faces an output-inflation trade-off, and is only constrained efficient under a special parametric condition stating that the trade elasticity is identically equal to the product of the degree of home bias and the IES. To our knowledge, this condition has not received any attention in the literature.

Second, the direction of the inefficiency can be signed in two leading special cases extensively studied in the literature. The first one, which the early NOEM literature almost exclusively focused on (e.g., Clarida et al. 2002 or Benigno and Benigno 2003), home bias is abstracted from ($\alpha = 1/2$) and purchasing power parity (PPP) holds. In that case, relative demand should be distorted away from the region with the most negative output gap, $\theta_t = 2y_t^D$ and the high-inflation region over-borrows, as the optimal tax differential satisfies $\tau_t^D = \sigma\eta\pi_t^D$. The second one, commonly referred to as Cole and Obstfeld (1991) preferences and popularized in the NOEM literature by Corsetti and Pesenti (2001), features unitary elasticities ($\sigma = \eta = 1$) and makes the two countries insular.[16] In this case too, the optimal relative demand gap is proportional to the cross-country difference in the output gap, $\theta_t = y_t^D/(1-\alpha)$, and the high-inflation region over-borrows, $\tau_t^D = \pi_t^D$. This later observation is noteworthy for at least two reasons. The first one is that under unitary elasticities, there is no cross-country borrowing under free capital mobility according to (9). Hence, there can be over-borrowing (or under-saving) even in the absence of any external imbalance. The second reason is that over-borrowing occurs despite the absence of any monetary spillovers under free capital mobility. Since the constrained efficient allocation does features spillover, our results indicate that spillovers can be good (i.e., are not necessarily a symptom of inefficient outcomes).

Third, and more generally, the scenarios just discussed, where the relative demand imbalance is proportional to the cross-country difference in the output gap and the high-inflation country over-borrows, appear to be the most relevant ones empirically. Indeed, most calibrations of the model place the trade elasticity above one and the IES below one, in which case the condition $\sigma\chi > (1-2\alpha)$ is necessarily satisfied.

What are the mechanisms behind these over- and under-borrowing results? We next argue that they have to do with a *pecuniary externality* operating via firms' marginal costs when monetary policy faces an output-inflation trade-off.

[16]The insularity refers to the absence of international spillovers on output and inflation, although there are spillovers on consumption.

3.4 Externality via firms' marginal costs

To nail down the inefficiencies at work in the free capital mobility regime, it is useful to ask how a marginal deviation from the equilibrium external borrowing positions in that regime alter the constraints faced by monetary policy and hence aggregate outcomes.

Consider a marginal increase in borrowing by Home from Foreign at instant t (i.e., $\theta_t = \epsilon$ for some small ϵ, leaving $\theta_k = 0$ for all other $k \neq t$).[17] Substituting for the equilibrium terms of trade using (15) into the marginal cost expressions (12a)-(12b) and applying the envelope theorem, the change in the loss function induced by this perturbation is given by

$$\frac{dL_t}{d\theta_t} = \varphi_t^D \frac{\partial mc^D(y_t^D, \theta_t)}{\partial \theta_t}. \tag{21}$$

The marginal increase in borrowing by Home from Foreign affects global welfare losses via its effects on the cross-country "difference" in marginal costs. Now, observe that the cross-country difference in marginal costs can be decomposed into two components

$$mc^D(y_t^D, \theta_t) = \underbrace{\left[\phi + \frac{1-2\alpha}{\omega}\sigma\right] y_t^D + \frac{\alpha\sigma^2}{\omega}\chi\theta_t}_{\text{difference in real wages}} + \underbrace{\frac{\alpha\sigma}{\omega}\left[2y_t^D - (1-2\alpha)\theta_t\right]}_{\text{differences in purchasing power}} + u_t^D. \tag{22}$$

The first component reflects cross-country differences in labor costs arising from differences in the real wage (in terms of each country's consumption bundle). The second term reflects cross-country differences in purchasing power arising from movements in the terms of trade. The marginal cost derivative in (21) is therefore given by

$$\frac{\partial mc^D(y_t^D, \theta_t)}{\partial \theta_t} = \frac{\alpha\sigma}{\omega}\left[\underbrace{\sigma\chi}_{\text{real wage effect}} - \underbrace{(1-2\alpha)}_{\text{purchasing power effect}}\right]. \tag{23}$$

First, raising Home consumption and lowering Foreign consumption shifts up labor supply in Home while shifting it down in Foreign. In equilibrium, this leads to a rise in Home's real wage and a drop in Foreign's real wage, thereby raising the cross-country difference in marginal costs. The strength of this effect corresponds to the trade elasticity normalized by the IES $\sigma\chi$. Second, the appreciation of the terms of trade, that follows

[17] For the sake of the argument, we assume that this increase in borrowing is compensated by a change in the date 0 implicit transfer. More generally, what matters for the externality to matter is that the balancing transaction occurs at a time when the government's multiplier on the NKPC (14) has a value different from the one at time t.

the increase in borrowing by Home from Foreign, raises Home households' purchasing power while decreasing Foreign households' purchasing power. In the presence of home bias in preferences $\alpha < 1/2$, this raises the marginal costs in Home and lowers them in Foreign hence reducing the cross-country difference. With no home bias $\alpha = 1/2$, because households in each country consume the same basket the cross-country difference in marginal costs is unaffected. The strength of this second effect corresponds to the degree of home bias $1 - 2\alpha$.

Hence, absent home bias in preference or when the trade elasticity is relatively high, $\sigma\chi > (1 - 2\alpha)$, the real wage effect dominates and an increase in θ_t raises the cross-country difference in marginal costs. When households exhibit large home bias in preferences and the trade elasticity is very low, $\sigma\chi < (1 - 2\alpha)$, the purchasing power effect dominates and the increase in θ_t lowers the cross-country difference in marginal costs. The two effects cancel out if and only if $\sigma\chi = (1 - 2\alpha)$, in which case the difference in marginal costs independent of θ_t.

These effects of marginal changes in external borrowing work in general equilibrium as prices adjust in goods and labor markets. As a result, they are ignored by atomistic agents. Yet, when the output-inflation trade-off is more stringent in one of the two countries, i.e. when $\varphi_t^D \neq 0$, a marginal increase in borrowing by Home from Foreign at instant t has a first-order welfare effect by tightening or relaxing the constraint faced by the monetary authority, as indicated by (21).

We next argue that the externalities just discussed could be so powerful that they may well result in trade imbalances of opposite signs in the constrained efficient regime and in the free capital mobility regime.

3.5 Topsy-turvy capital flows

Combining the targeting rule (19) with the equilibrium terms of trade expression (15), we obtain that the terms of trade is proportional to the cross-country output gap in the constrained inefficient regime, $s_t = 2y_t^D/\chi$, albeit with a different coefficient than under free capital mobility, where the relationship follows from (15) and is given by $s_t = 2\sigma y_t^D/\omega$. Substituting these terms of trade expressions into the net export expression (9), we obtain a trade balance of

$$nx_t = -\frac{2\alpha}{\sigma\chi}\, y_t^D \tag{24}$$

under the constrained efficient regime, while the trade balance under free capital mobility is given by:

$$nx_t = \frac{\omega - 1}{\omega} y_t^D. \tag{25}$$

This indicates qualitatively different trade imbalance patterns under the two regimes, which we summarize in the following proposition.

Proposition 3 (Topsy-turvy capital flows)**.** *In the constrained efficient capital account regime, the country with the most negative output gap always runs a trade surplus. This contrasts with the free capital mobility regime, where the country with the most negative output gap runs a trade deficit if $\sigma\eta > 1$ and a trade surplus if $\sigma\eta < 1$.*

Proof. The proof follows directly from (24), (25), and the definitions of ω and χ. □

The proposition implies that in the presence of cross-country differences in the severity of (policy induced) recession, capital flows are *topsy-turvy* under free capital mobility in the empirically plausible case where $\sigma\eta > 1$. That is, the country with the deepest recession runs a trade deficit while it should be running a trade surplus. Hence, rather than simply causing excessive borrowing, the pecuniary externality discussed in Section 3.4 is likely to be strong enough to flip the direction of capital flows.

Neoclassical and Keynesian motives of inter-temporal trade. To understand the essence of Proposition 3, it is useful to decipher the various motives for inter-temporal trade in the model. In the free capital mobility regime, these motives are purely neoclassical and are well understood since at least Cole and Obstfeld (1991): A temporarily lower income in Home creates an incentive to borrow, but the terms of trade appreciation accompanying this lower income generates an incentive to save. When the intra-temporal elasticity is high (i.e. $\sigma\eta > 1$), terms of trade movements are mild, and the first effect dominates. When the intra-temporal elasticity is low (i.e. $\sigma\eta < 1$), terms of trade movements are strong, and the second effect dominates. And when the intra- and inter-temporal elasticities are equal, the two effects neutralize each other.

In the constrained efficient regime, an additional *Keynesian macroeconomic stabilization motive*, capturing the externality outlined above, is also present. This motive calls for relaxing the output-inflation trade-off in the country where it is the least favorable. For the sake of illustrating the scope for topsy-turvy capital flows, consider the case of the Cole-Obstfeld parameter specification (or any other case satisfying $\sigma\eta = 1$). As we just argued, in this case the two neoclassical motives cancel out and result is zero trade imbalances under free capital mobility. In the constrained efficient capital flow regime, the Keynesian motive in addition generates an incentive to save for the country with the lowest output, as $\sigma\chi > (1-2\alpha)$ holds (see Proposition 2 and discussion of Section 3.4). That country thus experiences a trade surplus. For $\sigma\eta$ slightly above one, the net neoclassical effect becomes slightly positive, generating a trade deficit by the country with the most negative output gap under free capitail mobility, but the Keynesian effect dominates to yield a trade surplus by that country in the constrained efficient regime. As $\sigma\eta$ is raised further, the net neoclassical effect grows stronger in both regime, but it never overturns the Keynesian effect in the constrained efficient regime.[18]

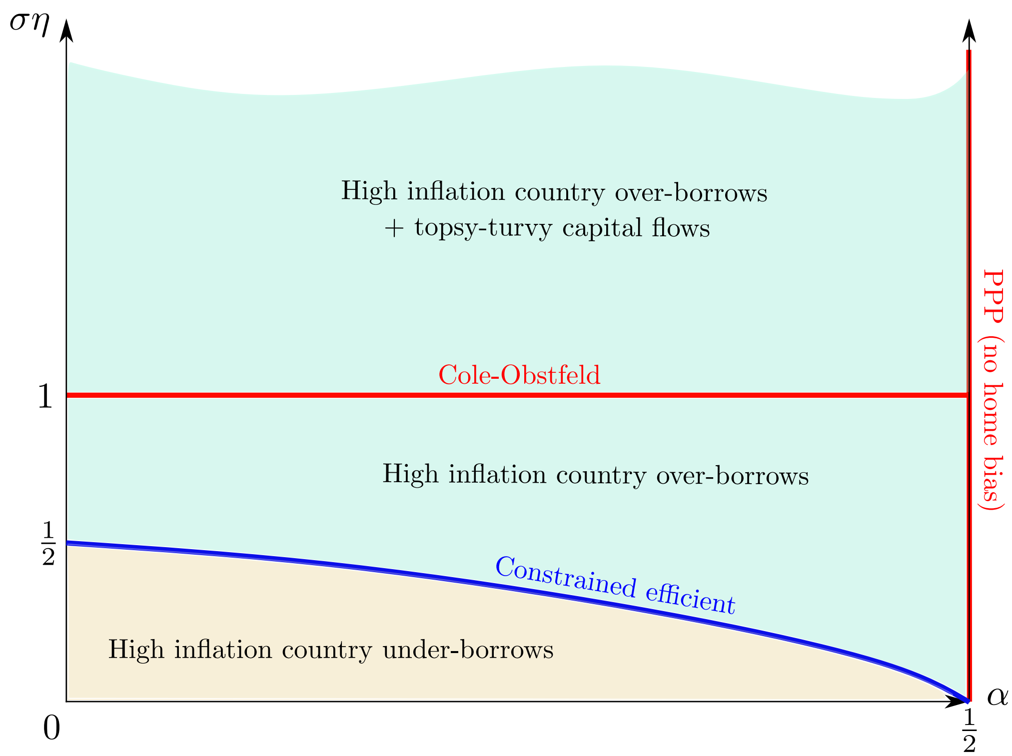


Figure 1: Characterization of capital flows in free capital mobility regime.

This topsy-turvy capital flows result is represented graphically in Figure 1, together with the over- vs under-borrowoing result of Corrolary 2. For any set of parameters, the figure indicates, based on the value of α and $\sigma\eta$, whether the free capital mobility regime is constrained efficient (blue concave curve), whether in that regime the high-inflation country over-borrows (area above the constrained efficient curve) or under-borrows (area

[18]In the limit where $\sigma\eta \to \infty$, as long as $\alpha > 0$, the trade balance becomes proportional to the difference in the output gap under free capital mobility, $nx_t = y_t^D$, but converges to zero in the constrained efficient regime, $nx_t = 0$.

below the constrained efficient curve), and whether that regime features topsy-turvy capital flows (area above the red Cole-Obstfeld parametrization line).

Is capital mobility harmful *per se*? Of course not. Rather, what our analysis uncovers is that prices in unregulated international financial markets do not accurately reflect the social value of external borrowing in the presence of an output-inflation trade-off, and that wrong price signals can result in capital flows patterns remarkably at odds with those that would be desirable from a social perspective. But they do not indicate that capital mobility is bad, nor that capital flow volatility is excessive in response to shocks generating policy trade-offs. In fact, it is easy to see that in some cases, such as when $\sigma\eta = 1$, capital flows are larger under the constrained efficient regime than under the free capital mobility regime.

4 External adjustment to cost-push shocks

To make the uncovered inefficiencies more tangible, we now look at the world economy's adjustment to an unanticipated temporary cost-push (supply) shock that gives rise to an output-inflation trade-off of unequal stringency in the two regions.

Cost-push shock scenario. For concreteness, suppose that Home is subject to an inflationary cost-push shock such that $u_t = 2\bar{u} > 0$ for some $\bar{u} > 0$ for $t \in [0, T)$ and $u_t = 0$ for $t \geq T$, while Foreign is not hit by any shock (i.e., $u_t^* = 0$ for $t \geq 0$). In terms of the "world" and "difference" shocks appearing in (13) and (14), we therefore have

$$u_t^W = u_t^D = \begin{cases} \bar{u} > 0 & \text{for} \quad t \in [0, T) \\ 0 & \text{for} \quad t \geq T. \end{cases} \tag{26}$$

As is well understood, monetary policy will not able to perfectly stabilize all variables under this scenario. Instead, it will trade off output gap and inflation distortions, as emphasized in Section 3.2. The main advantage of the step-function scenario in (26) is to allow a sharp graphical characterization of this adjustment under the two alternative capital account regimes.

4.1 Adjustment under free capital mobility

In a free capital mobility regime, $\theta_t = 0\ \forall t \geq 0$. Accounting for this fact when substituting the equilibrium terms of trade expression (15) into the NKPC in difference (14) yields a dynamic equation for the cross-country difference in inflation as a function of itself and the difference in the output gap:

$$\dot{\pi}_t^D = \rho \pi_t^D - \kappa \left(\frac{\sigma}{\omega} + \phi \right) y_t^D - \kappa u_t^D. \tag{27}$$

Meanwhile, differentiating the targeting rule (18) with respect to time yields a dynamic equation for the cross-country difference in the output gap as a function of the cross-country difference in inflation:

$$\dot{y}_t^D = -\varepsilon \pi_t^D. \tag{28}$$

(27) and (28) form a dynamical system in π_t^D and y_t^D whose solution encapsulates the dynamics of the cross-country block of the model. π_t^D is a jump variable, and although y_t^D could in principle jump, under the optimal plan it is predetermined at $y_0^D = 0$.[19] The system is thus saddle-path stable and the solution can be conveniently represented with the help of a phase diagram. The $\dot{y}_t^D = 0$ locus is described by $\pi_t^D = 0$, while the $\dot{\pi}_t^D = 0$ locus is described by $\rho \pi_t^D = \kappa \left(\frac{\sigma}{\omega} + \phi \right) y_t^D + \kappa u_t^D$. Given our shock scenario, in the (y_t^D, π_t^D) space, the $\dot{y}_t^D = 0$ locus is therefore always a flat line at 0, while the $\dot{\pi}_t^D = 0$ locus is an upward sloping straight line with slope $\kappa \left(\frac{\sigma}{\omega} + \phi \right) / \rho$ and intercept $\kappa \bar{u} / \rho > 0$ in the short-run (i.e., for $t \in [0, T)$) and intercept 0 in the long-run (i.e., for $t \geq T$).

The loci are represented in Figure 2, where y_t^D rises (diminishes) south (north) of the $\dot{y}_t^D = 0$ locus and π_t^D rises (diminishes) west (east) of the $\dot{\pi}_t^D = 0$ locus. The fictional saddle-path associated with the system being permanently governed by the short-term loci is represented by the upper dashed upward sloping line, while that associated with the system being permanently governed by the long-term loci is represented by the lower dashed upward sloping line. The actual saddle path is represented by the thick curve with arrows.

The inflationary cost-push shock in Home naturally drives a cross-country difference in inflation up on impact. But the initial jump in the inflation difference is limited by monetary policy's commitment to generate a more negative output gap in Home than in Foreign in the future, with the difference in the output gap displaying a hump shape. To

[19] The co-state variable φ_t^D is backward looking with an initial condition $\varphi_0^D = 0$, and both y_t^D and s_t are proportional to φ_t^D (see equations (A.20), (A.24) and (A.25) with $\theta_t = 0\ \forall t$).

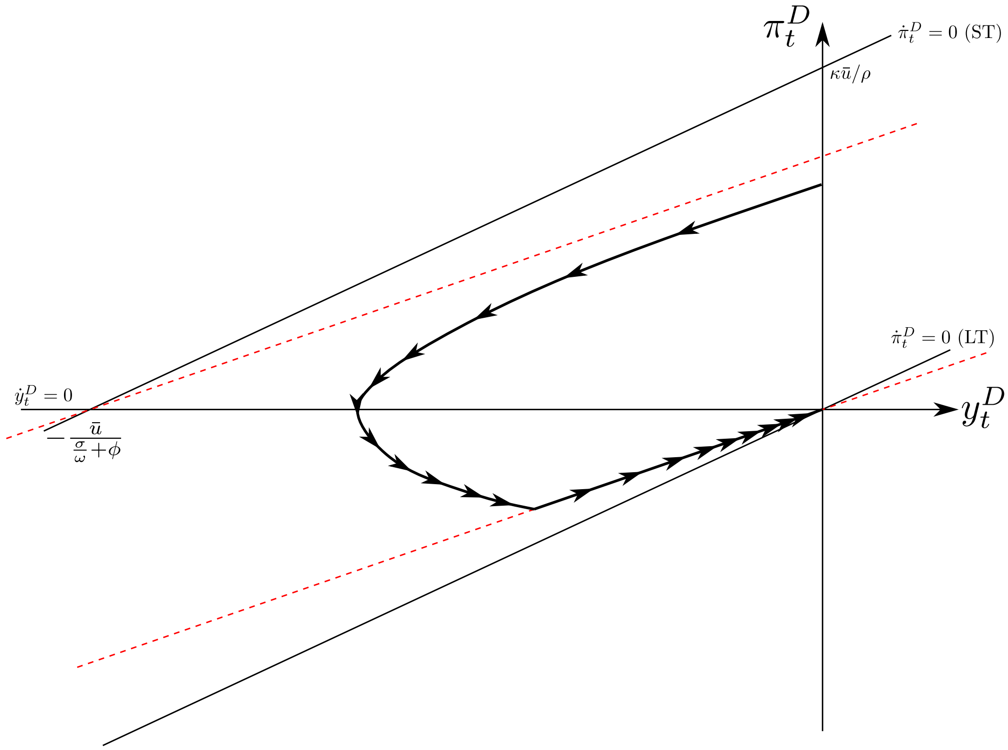


Figure 2: Output-inflation trade-off under free capital mobility.
Note: (ST) denotes short-term $\dot{\pi}_t^D = 0$ locus, (LT) denotes long-term $\dot{\pi}_t^D = 0$ locus.

support this path for the output gap differential, the terms of trade gap needs to follow a similar hump shape, indicating persistently (misaligned and) appreciated terms of trade throughout the episode.

Regarding cross-border capital flows, several patterns can emergence. From (25), a hump-shaped trade deficit arises if $\sigma\eta > 1$, while a hump-shaped trade surplus arises if $\sigma\eta < 1$. When $\sigma\eta = 1$, trade remains balance in response to the cost-push shock.

4.2 Adjustment under constrained efficient regime

In the constrained efficient regime, the path of θ_t satisfies the targeting rule (19). Accounting for this fact when substituting the equilibrium terms of trade expression (15) into the NKPC in difference (14) again yields a dynamic equation for the cross-country difference in inflation as a function of itself and the difference in the output gap:

$$\dot{\pi}_t^D = \rho\pi_t^D - \kappa\left[\frac{\sigma}{\omega} + \phi + \frac{2\alpha}{\chi\omega}\left(\sigma\chi - (1-2\alpha)\right)^2\right] y_t^D - \kappa u_t^D \tag{29}$$

where the last term in the square bracket reflects the optimal management of the relative demand gap. This term is non-negative, and equal to zero only in the knife-edge case where $\sigma\chi = (1-2\alpha)$. (29) and (27) now form the dynamical system in π_t^D and y_t^D whose solution represents the dynamics of the cross-country block of the model. Again, π_t^D is a jump variable, and y_t^D is predetermined at $y_0^D = 0$ under the optimal plan. The system is again saddle-path stable and is represented with a phase diagram in Figure 3.

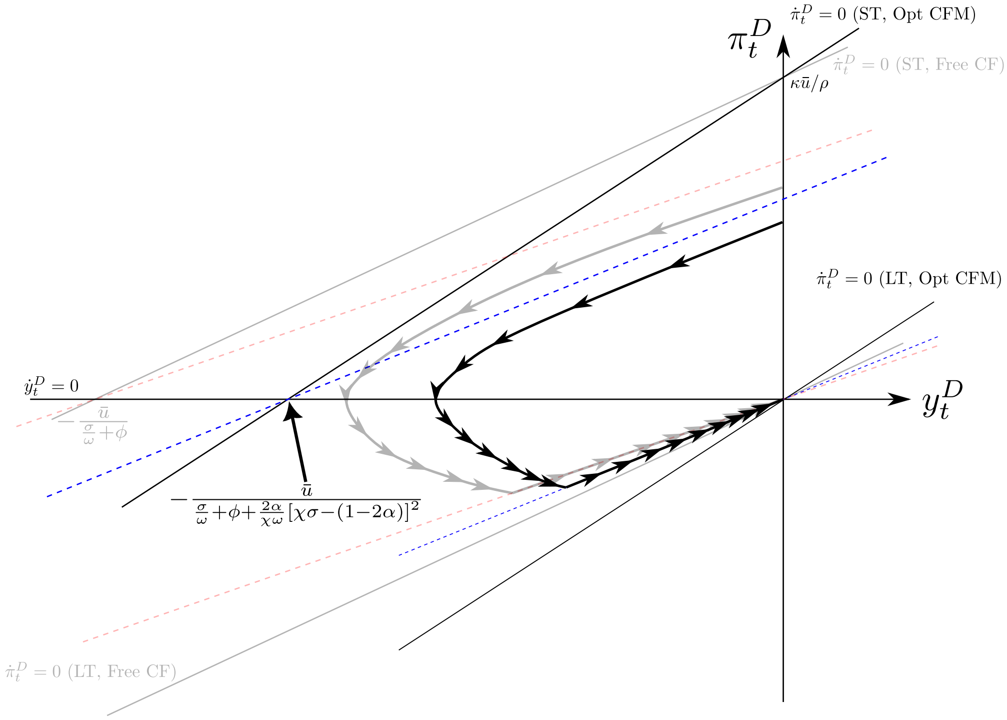
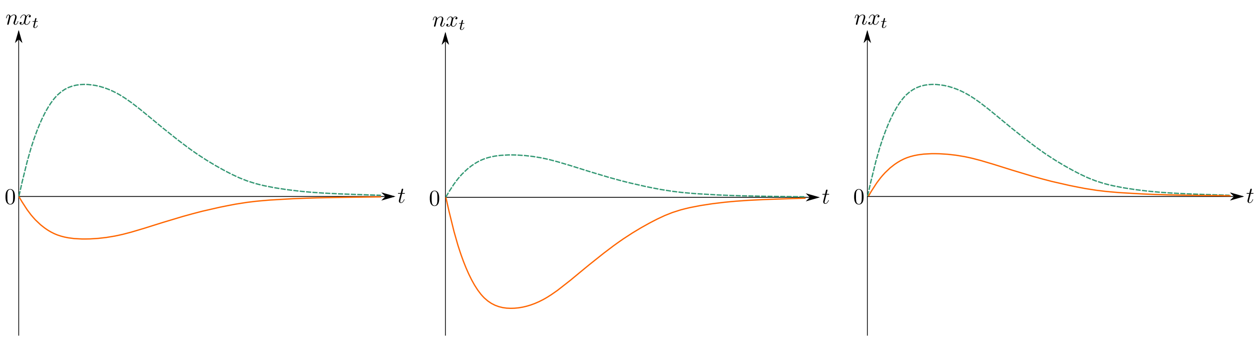


Figure 3: Output-inflation trade-off in the constrained efficient capital flow regime.
Note: (ST) denotes short-term $\dot{\pi}_t^D = 0$ locus, (LT) denotes long-term $\dot{\pi}_t^D = 0$ locus.

As under free capital mobility, the $\dot{y}_t^D = 0$ locus is described by $\pi_t^D = 0$. But this time, the $\dot{\pi}_t^D = 0$ locus is described by $\rho\pi_t^D = \kappa\left[\frac{\sigma}{\omega} + \phi + \frac{2\alpha}{\chi\omega}(\sigma\chi - (1-2\alpha))^2\right] y_t^D + \kappa u_t^D$. The only difference with the phase diagram of Figure 2 is that the $\dot{\pi}_t^D = 0$ locus now has a (weakly) steeper slope of $\kappa\left[\frac{\sigma}{\omega} + \phi + \frac{2\alpha}{\chi\omega}(\sigma\chi - (1-2\alpha))^2\right]/\rho$ in the short-run. This slope is strictly steeper, except in the knife-edge cases where $\sigma\chi = (1-2\alpha)$ in which case the two phase diagrams coincide. The phase diagram shows that the constrained efficient capital flow regime results in a more favorable trade-off between the stabilization of the cross-country difference in the output gap and the cross-country difference in domestic inflation, regardless of the direction of the inefficiency.

Over-borrowing and excessive capital flow volatility? How do capital flows differ across the two regimes? The possibility of topsy-turvy capital flows in our model means that unlike in other theories predicting over-borrowing, trade-imbalances may be less or more volatile under the constrained efficient regime than under free capital mobility. Focusing on parameter configurations generating over-borrowing (i.e., for $\sigma\chi > (1 - 2\alpha)$), Figure 4 represents three distinct scenarios. In panels (a) and (b), following the shock, Home runs a trade deficit under free capital mobility but a trade surplus under the constrained efficient regime. This is the case where capital flows are topsy turvy. However, in the case of panel (a), where $\sigma\eta$ is only slightly above unity, capital flows are insufficiently volatile under free capital mobility, while in the case of panel (b), where $\sigma\eta$ is larger, they are excessively volatile. In panel (c), $\sigma\eta$ is slightly below unity so capital flows are not topsy turvy. In this case Home runs trade surpluses following the shock under both regimes, albeit larger ones in the constrained efficient regime.



(a) Too little volatility ($\sigma\eta > 1$) (b) Too much volatility ($\sigma\eta > 1$) (c) Too little volatility ($\sigma\eta < 1$)

Figure 4: Possible capital flow patterns following cost-push shocks.
Note: Solid lines represent free capital mobility regime, and dashed lines represent constrained efficient capital flow regime.

Given the variety of different scenarios potentially predicted by our model (i.e., excessively or insufficiently volatile capital flows), it seems relevant to take a closer look at a calibration commonly used in the literature.

4.3 Calibrated example

To further illustrate how the macroeconomic adjustments play out under both regimes, we now turn to presenting impulse response functions to an asymmetric cost-push shock in a calibrated version of the model. To do so, we draw heavily on the calibration used by Groll and Monacelli (2020) to study impulse responses to cost-push shocks. Rather than assuming a step function as in Sections 4.2 and 4.1, we consider a more standard mean reverting shock. In particular, we hit the economy with a cost-push shock of 10% that mean reverts at rate 0.42 per year, yielding an annual autocorrelation of 0.65, or equivalently, a quarterly autocorrelation of 0.9. The parameter σ is set to 1, implying an IES of 1, and the labor supply elasticity parameter ϕ is set to zero. The home bias parameter, α, is set to 0.25, which implies a weight of 0.75 on domestically produced goods in the consumption basket. The trade elasticity χ plays an important role for our results, as it determines the direction of the inefficiency and the scope for topsy-turvy capital flows, with high elasticities making over-borrowing and topsy-turvy capital flows more likely. Simonovska and Waugh (2014) report a range of trade elasticity estimates from 2.69 to 4.47. We conservatively set χ near the lower bound of this range to $\chi = 3$, which implies an elasticity of substitution between domestic and foreign good η of 2. The discount rate parameter, ρ, and the parameter for the probability of adjustment of nominal prices, ρ_δ, are both set to standard values: $\rho = 0.04$ and $\rho_\delta = 1 - 0.75^4$. Finally, the elasticity of substitution among differentiated intermediate goods, ε, is set to 7.66, corresponding to a 15% net markup. All parameters are hence set to the same value as in Groll and Monacelli (2020) (adjusting for our annual frequency).

Figure 5 illustrates the difference in the response of macroeconomic variables to the cost-push shock under free capital mobility vs. the constrained efficient capital flow regime. It is well understood that under free capital mobility, the efficient allocation cannot be achieved following cost-push shocks. To limit PPI inflation in Home following the shock, monetary policy commits to negative output gaps in the future. The monetary policy response entails a positive spillover in Foreign, where the positive output gap nearly reaches 3% and a terms of trade appreciation peaking at 9%. Home runs a trade deficit of up to 3.5% of GDP.

In the optimal capital flow management regime, opening a demand gap in favor of Foreign helps reduce the magnitude of the Home output gap and mitigate international relative price misalignment at the expense of distorting the international risk-sharing condition. The Home output gap drops to -10% rather than -13% and the terms of trade appreciates by no more than 3%. The negative demand imbalance redirects demand toward Foreign, but the significantly smaller terms of trade appreciation results in a mildly negative Foreign output gap (rather than a positive one under free capital mobility). As a result, PPI inflation in Foreign is also more stable. Therefore, it is not a zero-sum game and both countries achieve a superior stabilization of output and inflation.

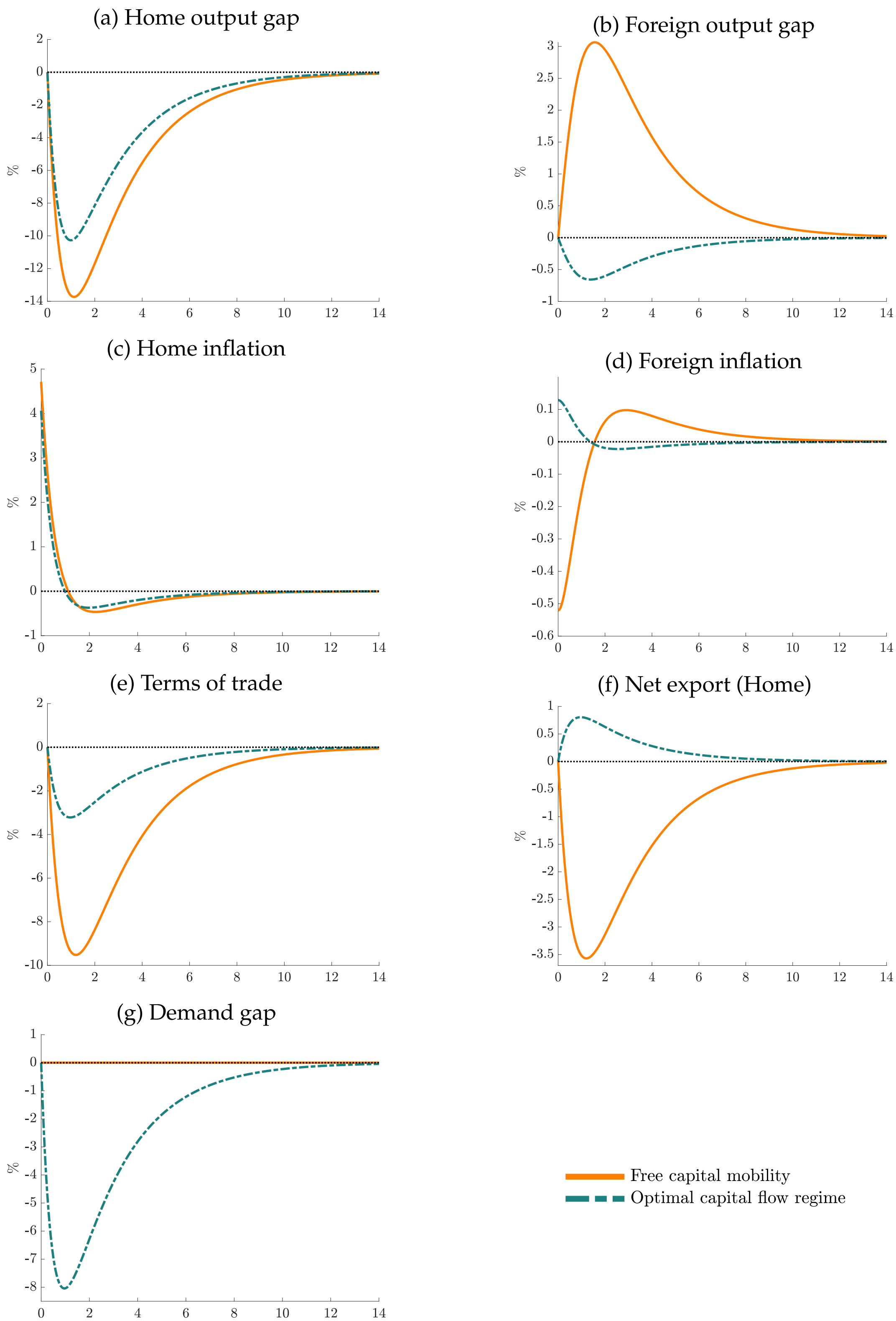


Figure 5: Impulse responses to cost-push shock in Home.
Note: Solid lines represent free capital mobility regime, and dashed lines represent constrained efficient capital flow regime.

5 Conclusion

We point to a pecuniary externality operating via firms' marginal costs in open-economy models with nominal rigidities. For plausible values of elasticities, the externality causes over-borrowing by high-inflation regions and capital to flow in the wrong direction during stagflation episodes: while a constrained efficient regime would require outflows from the regions with the most negative output gap, a free capital mobility regime features capital inflows into such regions. Our results cast doubts on the classical view that free capital mobility promotes macroeconomic adjustment, in particular in a stagflation context.

Our results have implications beyond open-economy macroeconomics. Indeed, the insight that financial decisions exert externalities on policy trade-offs through the supply side ought to apply more generally to other heterogeneous agents, multi-sector macroeconomic models with nominal rigidities. Given the rising popularity of heterogeneous agents New Keynesian (HANK) models and current concerns about the possibility of stagflation, the study of such externalities appears to be a pressing issue for future research.

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APPENDIX TO "STAGFLATION AND TOPSY TURVY CAPITAL FLOWS"

A Derivation of the loss function

The goal of the global planner is to maximize the average welfare function of Home and Foreign households. In this section, we rewrite the objective function in terms of the squared output gap, squared inflation and squared terms-of-trade and relative demand gap. Note that the period utility of the global planner is

$$v_t \equiv \frac{1}{2}\left[\frac{1}{1-\sigma}(C_t)^{1-\sigma} - \frac{1}{1+\phi}(N_t)^{1+\phi}\right] + \frac{1}{2}\left[\frac{1}{1-\sigma}(C_t^*)^{1-\sigma} - \frac{1}{1+\phi}(N_t^*)^{1+\phi}\right]$$

The loss relative to the efficient outcome is then $v_t - v_t^{max}$ where v^{max} is the maximized welfare that is welfare when C_t, C_t^*, N_t and N_t^* take on their efficient values. We start by describing the efficient allocation then turn to deriving the second order approximation of the loss function.

Efficient allocation. The socially optimal allocation solves the following static problem

$$\max_{C_{H,t},C_{H,t}^*,C_{F,t},C_{F,t}^*,N_t,N_t^*} \frac{1}{1-\sigma}\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta(1-\sigma)}{\eta-1}} - \frac{1}{1+\phi}(N_t)^{1+\phi}$$
$$+\frac{1}{1-\sigma}\left[(1-\alpha)^{\frac{1}{\eta}}(C_{F,t}^*)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{H,t}^*)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta(1-\sigma)}{\eta-1}} - \frac{(N_t^*)^{1+\phi}}{1+\phi}$$

subject to

$$C_{H,t} + C_{H,t}^* = N_t \tag{A.1}$$
$$C_{F,t} + C_{F,t}^* = N_t^* \tag{A.2}$$

Let $\vartheta_{H,t}$ and $\vartheta_{F,t}$ denote the multipliers on (A.1) and (A.2). The first order conditions are

$$[C_{H,t}] :: \quad \vartheta_{H,t} = (1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{-\frac{1}{\eta}}(C_t)^{\frac{1}{\eta}-\sigma} \tag{A.3a}$$
$$[C_{F,t}] :: \quad \vartheta_{F,t}^* = \alpha^{\frac{1}{\eta}}(C_{F,t})^{-\frac{1}{\eta}}(C_t)^{\frac{1}{\eta}-\sigma} \tag{A.3b}$$
$$[C_{H,t}^*] :: \quad \vartheta_{H,t} = \alpha^{\frac{1}{\eta}}(C_{H,t}^*)^{-\frac{1}{\eta}}(C_t^*)^{\frac{1}{\eta}-\sigma} \tag{A.4a}$$
$$[C_{F,t}^*] :: \quad \vartheta_{F,t}j = (1-\alpha)^{\frac{1}{\eta}}(C_{F,t}^*)^{-\frac{1}{\eta}}(C_t^*)^{\frac{1}{\eta}-\sigma} \tag{A.4b}$$

$$[N_t] :: \quad (N_t)^{\phi} = \vartheta_{H,t} \tag{A.5a}$$

$$[N_t^*] :: \quad (N_t^*)^{\phi} = \vartheta_{F,t}^* \tag{A.5b}$$

Combining (A.3a) and (A.3b) after multiplying the first equation by $C^{H,t}\cdot$ and the second $C^{F,t}\cdot$ and proceeding similar with (A.4a) and (A.4b), we arrive to

$$\vartheta_{H,t}C_{H,t} + \vartheta_{F,t}^* C_{F,t} = (C_t)^{1-\sigma} + (C_t^*)^{1-\sigma} \tag{A.6a}$$

$$\vartheta_{H,t}C_{H,t}^* + \vartheta_{F,t}C_{F,t}^* = (C_t)^{1-\sigma} + (C_t^*)^{1-\sigma} \tag{A.6b}$$

Substituting the resource constraint into (A.5a) and (A.5b) yields $(N_t)^{1+\phi} + (N_t^*)^{1+\phi} = \vartheta_{H,t}(C_{H,t} + C_{H,t}^*) + \vartheta_{F,t}^*(C_{F,t} + C_{F,t}^*)$ which combined with (A.6a) and (A.6b) leads to

$$(C_t)^{1-\sigma} + (C_t^*)^{1-\sigma} = (N_t)^{1+\phi} + (N_t^*)^{1+\phi} \tag{A.7}$$

From market clearing and symmetry $\bar{C}_t = \bar{C}_t^* = \bar{N}_t = \bar{N}_t^* = 1$ where variables with a *bar* denote efficient values. It is also It is straightforward to see that $\bar{C}_{H,t} = \bar{C}_{F,t}^* = 1 - \alpha$ and $\bar{C}_{F,t} = \bar{C}_{H,t}^* = \alpha$. In log-deviations, we get

$$\bar{c}_{H,t} = \bar{c}_{H,t}^* = \bar{c}_{F,t} = \bar{c}_{F,t}^* = 0 \quad \text{and} \quad \bar{n}_t = \bar{n}_t^* = 0. \tag{A.8}$$

Loss function. The second order approximation of the period utility around the efficient allocation (using $\bar{C}^{1-\sigma} = \bar{N}^{1+\phi}$ from (A.7) and symmetry) is given by

$$v_t = \frac{\sigma+\phi}{1+\phi}\frac{(\bar{C})^{1-\sigma}}{1-\sigma} + \frac{(\bar{C})^{1-\sigma}}{2}\Big[(c_t + c_t^*) + \frac{1-\sigma}{2}\left((c_t)^2 + (c_t^*)^2\right) - (n_t + n_t^*) - \frac{1+\phi}{2}\left((n_t)^2 + (n_t^*)^2\right) + o\left(||u||^3\right)\Big] \tag{A.9}$$

where $+o\left(||u||^3\right)$ indicate the 3^{rd} and higher order terms left out. Note from (A.8) and (A.9) that $v_t^{max} = \frac{\sigma+\phi}{1+\phi}\frac{(\bar{C})^{1-\sigma}}{1-\sigma}$. The period loss function loss function is then

$$v - v_t^{max} = \frac{1}{2}\Big[(c_t + c_t^*) + \frac{1-\sigma}{2}\left((c_t)^2 + (c_t^*)^2\right) - (n_t + n_t^*) - \frac{1+\phi}{2}\left((n_t)^2 + (n_t^*)^2\right) + o\left(||u||^3\right)\Big] \tag{A.10}$$

We now need to use the second order approximation of the aggregate demand equations and aggregate employment to replace for c_t and n_t. First note that after substituting for the international risk sharing condition (4), the aggregate demand for home goods can be rewritten as

$$Y_t = \left[(1-\alpha) + \alpha\,(S_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha\Theta_t^{-1} Q_t^{\eta-\frac{1}{\sigma}}\right] C_t$$

Taking the second order approximation we get

$$\begin{aligned} y_t = c_t - \alpha\theta_t + \frac{\omega - (1-2\alpha)}{2\sigma} s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \frac{1}{2}\alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\sigma\eta - 1)\sigma^{-1} s_t\right]^2 + o(||u||^3) \end{aligned} \tag{A.11}$$

where $\omega = \sigma\eta + (\sigma\eta - 1)(1-2\alpha)^2$. Similarly, demand for foreign good is given by

$$Y_t^* = \left[(1-\alpha) + \alpha\,(S_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha\Theta_t Q_t^{\frac{1}{\sigma}-\eta}\right] C_t^*,$$

and the second order approximation is given by

$$\begin{aligned} y_t^* = c_t^* + \alpha\theta_t - \frac{\omega - (1-2\alpha)}{2\sigma} s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \frac{1}{2}\alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\sigma\eta - 1)\sigma^{-1} s_t\right]^2 + o(||u||^3) \end{aligned} \tag{A.12}$$

We then combine (A.11) and (A.12) to obtain

$$\begin{aligned} c_t + c_t^* = y_t + y_t^* + \alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\sigma\eta - 1)\sigma^{-1} s_t\right]^2 + o(||u||^3) \end{aligned} \tag{A.13}$$

Using again (A.11) and (A.12) and after some algebraic manipulation we get

$$\begin{aligned} (c_t)^2 + (c_t^*)^2 = (y_t)^2 + (y_t^*)^2 - 2\alpha(1-\alpha)\,(\sigma\eta)^2\left(\sigma^{-1} s_t\right)^2 \\ + 2\alpha(1-\alpha)\left(\theta_t - (\sigma\eta - 1)(1-2\alpha)\sigma^{-1} s_t\right)^2 + o(||u||^3) \end{aligned} \tag{A.14}$$

Aggregate employment is given $N_t = Y_t Z_t$ with $Z_t = \int_0^1 \left(P_{Ht(l)}/P_{Ht}\right)^{-\varepsilon} dl$. At the second order approximation $n_t = y_t + z_t + \frac{1}{2}y_t^2 + o(||u||^3)$ with $z_t = o + o(||u||^2)$. Thus, we have

$$n_t + n_t^* = y_t + y_t^* + \frac{1}{2}\left((y_t)^2 + (y_t^*)^2\right) + z_t + z_t^* + o(||u||^3) \tag{A.15}$$

$$(n_t)^2 + (n_t^*)^2 = (y_t)^2 + (y_t^*)^2 + o(||u||^3) \tag{A.16}$$

Plugging (A.13), (A.14), (A.15) and (A.16) into the (A.10) we obtain the following second order approximation of the period loss function

$$v - v_t^{max} = \frac{1}{2}\Big[z_t + z_t^* + (\sigma+\phi)(y_t)^2 + (\sigma+\phi)(y_t^*)^2 + 2\alpha(1-\alpha)(1-\eta\sigma)\eta(s_t)^2 \\ + 2\sigma\alpha(1-\alpha)\left(\theta_t - (\sigma\eta-1)(1-2\alpha)\sigma^{-1}s_t\right)^2\Big] + o(||u||^3)$$

The objective of the global planner is to minimize the loss function is $\mathbb{L} = \int_0^\infty e^{-\rho t}$. Using

$$\int_0^\infty e^{-\rho t} z_t dt = \int_0^\infty e^{-\rho t} \text{var}_l\left(P_{H,t}(l)\right) dt = \frac{1}{\kappa}\int_0^\infty e^{-\rho t}\left(\pi_{H,t}\right)^2 dt$$
$$\int_0^\infty e^{-\rho t} z_t^* dt = \int_0^\infty e^{-\rho t} \text{var}_l\left(P_{F,t}^*(l)\right) dt = \frac{1}{\kappa}\int_0^\infty e^{-\rho t}\left(\pi_{F,t}^*\right)^2 dt$$

and our definition of world and difference variables $(\pi_{H,t})^2 + (\pi_{F,t}^*)^2 = 2[(\pi_t^W)^2 + (\pi_t^D)^2]$ and $(y_t)^2 + (y_t^*)^2 = 2[(y_t^W)^2 + (y_t^D)^2]$ we arrive to

$$\mathbb{L} = \frac{1}{2}\int_0^\infty e^{-\rho t}\Big[2\frac{\varepsilon}{\kappa}\left((\pi_t^W)^2 + (\pi_t^D)^2\right)^2 + 2(\sigma+\phi)\left((y_t^W)^2 + (y_t^D)^2\right) \\ + 2\alpha(1-\alpha)(1-\eta\sigma)\eta(s_t)^2 + 2\sigma\alpha(1-\alpha)\left(\theta_t - (\sigma\eta-1)(1-2\alpha)\sigma^{-1}s_t\right)^2\Big] \tag{A.17}$$

which corresponds to (16).

B Optimal policy problem

We divide the loss (16) by a factor 2 since we can equivalently minimize a linear transformation of the objection function of the global planner. The optimal monetary policy problem is given by

$$\max_{\{\pi^W,\pi^D,x^W,y^D,s\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\Big[\frac{\varepsilon}{\kappa}\left((\pi_t^W)^2 + (\pi_t^D)^2\right) + (\sigma+\phi)\left((y_t^W)^2 + (y_t^D)^2\right) \\ + \alpha(1-\alpha)(1-\sigma\eta)\eta(s_t)^2 + \sigma\alpha(1-\alpha)\left(\theta_t - (\sigma\eta-1)(1-2\alpha)\sigma^{-1}s_t\right)^2\Big]$$

subject to

$$\dot{\pi}_t^W = \rho\pi_t^W - \kappa(\sigma+\phi)y_t^W - \kappa u_t^W \tag{A.18}$$
$$\dot{\pi}_t^D = \rho\pi_t^D - \kappa(\sigma+\phi)y_t^D + \kappa\frac{\omega-1}{2}s_t - \kappa\alpha\sigma\theta_t - \kappa u_t^D \tag{A.19}$$
$$2y_t^D = \omega\sigma^{-1}s_t + (1-2\alpha)\theta_t \tag{A.20}$$

Letting φ_t^W, φ_t^D, be the co-state associated with (A.18), (A.19), the first order conditions are

$$\left[\pi_t^W\right] :: \quad \dot{\varphi}_t^W = -\frac{\varepsilon}{\kappa}\pi_t^W \tag{A.21}$$
$$\left[\pi_t^D\right] :: \quad \dot{\varphi}_t^D = -\frac{\varepsilon}{\kappa}\pi_t^D \tag{A.22}$$
$$\left[y_t^W\right] :: \quad 0 = -(\sigma+\phi)y_t^W + \kappa(\sigma+\phi)\varphi_t^W \tag{A.23}$$
$$\left[y_t^D\right] :: \quad 0 = -(\sigma+\phi)y_t^D + \kappa(\sigma+\phi)\varphi_t^D - \Lambda_t \tag{A.24}$$
$$[s_t] :: \quad 0 = -(\omega-1)y_t^D + \kappa(\omega-1)\varphi_t^D - \omega\sigma^{-1}\Lambda_t \tag{A.25}$$
$$[\theta_t] :: \quad 0 = -\sigma\alpha(1-\alpha)\theta_t + \frac{\omega-1}{4}(1-2\alpha)s_t + \kappa\alpha\sigma\varphi_t^D + \frac{1}{2}(1-2\alpha)\Lambda_t \tag{A.26}$$

together with the initial conditions $\varphi_0^j = 0$ and transversality conditions $\lim_{t\to\infty} e^{-\rho t}\varphi_t^j = 0$ for $j \in \{W, D\}$ and where Λ_t is the Lagrange multiplier on (A.20).

B.1 Proof of Proposition 1

Combining (A.24) and (A.25) we have $\Lambda_t = 0$. Substituting this back into (A.24), we obtain

$$y_t^D - \kappa\varphi_t^D = 0. \tag{A.27}$$

Differentiating (A.23) and (A.27) with respect to time and noting from (A.21) and (A.22) that $\kappa\dot{\varphi}_t^W = -\varepsilon\pi_t^W$ and $\kappa\dot{\varphi}_t^D = -\varepsilon\pi_t^D$, we obtain

$$\begin{aligned} \dot{y}_t^W + \varepsilon\pi_t^W &= 0 \\ \dot{y}_t^D + \varepsilon\pi_t^D &= 0 \end{aligned} \tag{A.28}$$

From (A.23), $y_t^W = \kappa\varphi_t^W$, and given that $\varphi_0^W = 0$, we have $y_0^W = 0$. From (A.27) and (A.20) we have $y_0^D = 0$ and $2y_0^D + \omega s_0 = 0$ which imply that $y_0^D = s_0 = 0$. Thus, integrating between 0 and t we arrive to

$$y_t^W + \varepsilon(p_t^W - p_0^W) = 0 \tag{A.29}$$
$$y_t^D + \varepsilon(p_t^D - p_0^D) = 0 \tag{A.30}$$

B.2 Proof of Corollary 1

We consider the targeting rule (A.28) for world variables and differentiate this rule to obtain $\ddot{y}_t^W + \varepsilon\dot{\pi}_t^W = 0$. We then use (A.18), $\dot{\pi}_t^W = \rho\pi_t^W - \kappa(\sigma + \phi)y_t^W - \kappa u_t^W$, to substitute for $\dot{\pi}_t^W$ and obtain

$$\ddot{y}_t^W - \rho\dot{y}_t^W - \varepsilon\kappa(1+\phi)y_t^W = \varepsilon\kappa u_t^W \tag{A.31}$$

The polynomial characteristic of this equation has one negative eigenvalue $z_1 < 0$ and one positive eigenvalue $z_2 > 0$ where

$$z_1 = \frac{1}{2}\left(\rho - \sqrt{\rho^2 + 4\kappa\varepsilon(1+\phi)}\right) < 0 \quad \text{and} \quad z_2 = \frac{1}{2}\left(\rho + \sqrt{\rho^2 + 4\kappa\varepsilon(1+\phi)}\right) > 0$$

The solution of this second order differential equation takes the form

$$y_t^W = \vartheta_0 e^{z_1 t} + \vartheta_1 \int_0^t e^{z_1(t-s)} u_s^W ds + \vartheta_2 \int_t^\infty e^{z_2(t-s)} u_s^W ds. \tag{A.32}$$

Differentiating (A.32) and relating each term to (A.31) we obtain

$$\vartheta_1 = \vartheta_2 = -\frac{\varepsilon\kappa}{z_2 - z_1}.$$

Next, from (A.32) for $t = 0$ we get

$$\vartheta_0 = y_0^W + \frac{\varepsilon\kappa}{z_2 - z_1}\int_0^\infty e^{-z_2 s} u_s^W ds$$

From the initial condition for the co-state variable $\varphi_0^W = 0$, the relation $y_t^W = \kappa\varphi_t^W$ implies that $y_t^W = 0$. The solution to the optimal monetary policy problem is thus

$$y_t^W = -\frac{\varepsilon\kappa}{z_2 - z_1}\left[e^{z_1 t}\int_0^t \left(e^{-z_1 s} - e^{-z_2 s}\right) u_s^W ds + \left(e^{z_2 t} - e^{z_1 t}\right)\int_t^\infty e^{-z_2 s} u_s^W ds\right]. \tag{A.33}$$

Using (A.28), the path for the world inflation under the optimal monetary policy satisfies

$$\pi_t^W = \frac{\kappa}{z_2 - z_1}\left[z_1 e^{z_1 t}\int_0^t \left(e^{-z_1 s} - e^{-z_2 s}\right) u_s^W ds + \left(z_2 e^{z_2 t} - z_1 e^{z_1 t}\right)\int_t^\infty e^{-z_2 s} u_s^W ds\right]. \tag{A.34}$$

From (A.33) and (A.34), it follows that the paths of the world variables y_t^W and π_t^W are independent of the path of θ_t.

B.3 Proof of Proposition 2

Optimal capital flow management. Notice again that by combining (A.24) and (A.25) we obtain $\Lambda_t = 0$. Substituting it into the optimality condition (A.26) we arrive to

$$
\begin{aligned}
2\alpha(1-\alpha)\sigma\theta_t &= (1-2\alpha)\frac{\omega-1}{2}s_t + 2\kappa\sigma\alpha\varphi_t^D \\
&= (1-2\alpha)\frac{\omega-1}{2}s_t + 2\sigma\alpha y_t^D
\end{aligned}
\tag{A.35}
$$

where the second equality uses (A.27). We then plug equation (A.20) into (A.35) to substitute for y_t^D. We get

$$
\begin{aligned}
2\alpha(1-\alpha)\sigma\theta_t &= (1-2\alpha)\frac{\omega-1}{2}\left[\frac{2\sigma}{\omega}y_t^D - \left(\frac{1-2\alpha}{\omega}\right)\sigma\theta_t\right] + 2\sigma\alpha y_t^D \\
&= \frac{\omega-(1-2\alpha)}{\omega}\sigma y_t^D - (1-2\alpha)^2\frac{\omega-1}{2\omega}\sigma\theta_t
\end{aligned}
\tag{A.36}
$$

Rearranging the expression (A.36) leads to

$$
\begin{aligned}
\frac{1}{2\omega}\left[\omega-(1-2\alpha)^2\right]\theta_t &= \frac{\omega-(1-2\alpha)}{\omega}y_t^D \\
\frac{\alpha}{\omega}\left[\chi\sigma\right]\theta_t &= \frac{2\alpha}{\omega}\left[\chi\sigma-(1-2\alpha)\right]y_t^D
\end{aligned}
\tag{A.37}
$$

where we use $\omega = 2\alpha\chi\sigma - (1-2\alpha)^2$ to obtain the second equality (A.37). Finally, we simplify the above expression (A.37) and arrive to

$$
\theta_t = \frac{\chi\sigma-(1-2\alpha)}{\chi\sigma}2y_t^D. \tag{A.38}
$$

Taxes on capital flows. The optimal tax on capital flows follows directly from $\tau_t^D = \frac{1}{2}\sigma\dot{\theta}_t$. We start by rewriting θ_t in (A.38) as a function of s_t using (A.20). We have

$$
\theta_t = \frac{\chi\sigma-(1-2\alpha)}{\chi\sigma}\left[\frac{\omega}{\sigma}s_t + (1-2\alpha)\theta_t\right]. \tag{A.39}
$$

We differentiate and rearrange this expression to get $\omega\dot{\theta}_t = [\chi\sigma-(1-2\alpha)]\frac{\omega}{\sigma}\dot{s}_t$, where it worth noting again that $2\alpha\chi\sigma-(1-2\alpha)^2 = \omega$. Finally, we plug this expression into $\tau_t = \frac{1}{2}\sigma\dot{\theta}_t$ and use $\dot{s}_t = 2\pi_t^D$ to get

$$
\tau_t^D = \left[\chi\sigma-(1-2\alpha)\right]\pi_t^D. \tag{A.40}
$$